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Hideo Okawara's Mixed Signal Lecture Series

DSP-Based Testing – Fundamentals 18 Histogram Method in ADC Linearity Test

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Preface to the Series

ADC and DAC are the most typical mixed signal devices. In mixed signal testing, analog stimulus signal is generated by an arbitrary waveform generator (AWG) which employs a D/A converter inside, and analog signal is measured by a digitizer or a sampler which employs an A/D converter inside. The stimulus signal is created with mathematical method, and the measured signal is processed with mathematical method, extracting various parameters. It is based on digital signal processing (DSP) so that our test methodologies are often called DSP-based testing.

Test/application engineers in the mixed signal field should have thorough knowledge about DSP-based testing. FFT (Fast Fourier Transform) is the most powerful tool here. This corner will deliver a series of fundamental knowledge of DSP-based testing, especially FFT and its related topics. It will help test/application engineers comprehend what the DSP-based testing is and assorted techniques.

Editor's Note

For other articles in this series, please visit the Verigy web site at www.verigy.com/go/gosemi.

Histogram Method in ADC Linearity Test

Linearity is the most important specification of A/D converters (ADC). There are several methods available to test linearity of ADC. Histogram analysis is quite simple and easy to apply so that it is one of the most typical test methodologies. It may be called as a code density test. Ramp histogram and sine histogram tests have been used for a long time; however, linearity calculation equations for ramp/sine histogram are not well organized. In this document, histogram methods are discussed in detail, focusing on a terminal based (end-point) transfer function, easy-to-use equations are introduced with using cumulative distribution function for sine histogram. The method using this equation is free from overload level and offset of test signal sine so that test procedure is simple and fast in both hardware-wise and software-wise.

Transfer Function of A/D Converters

A transfer function of *n*-bit linear A/D converters (ADC) is depicted in Figure 1. The horizontal axis shows analog input level, and the vertical axis shows discrete code. Figure 1 (a) is an ideal transfer function, and (b) is an actual transfer function.

L_i and Lm_i ($i=0, 1, 2, \ldots, 2^n-2$) denote an ideal and an actual threshold levels of each code respectively. Q_i and Qm_i ($i=0, 1, 2, \ldots, 2^n-1$) denote an ideal and an actual quantization levels respectively. Quantization levels are the code centers. Then Q_i and Qm_i are described with threshold levels as below.

$$Q_i = \frac{L_{i-1} + L_i}{2} \quad (i=1, 2, 3, \ldots, 2^n-2) \qquad (1)$$

$$Qm_i = \frac{Lm_{i-1} + Lm_i}{2} \quad (i=1, 2, 3, \ldots, 2^n-2) \qquad (2)$$

In the case of terminal based or end-point transfer function, actual terminal quantization levels are identical to ideal levels as shown in Figure 1 (a) (b).

$$Qm_0 = Q_0 \qquad (3)$$

$$Qm_{2^n-1} = Q_{2^n-1} \qquad (4)$$

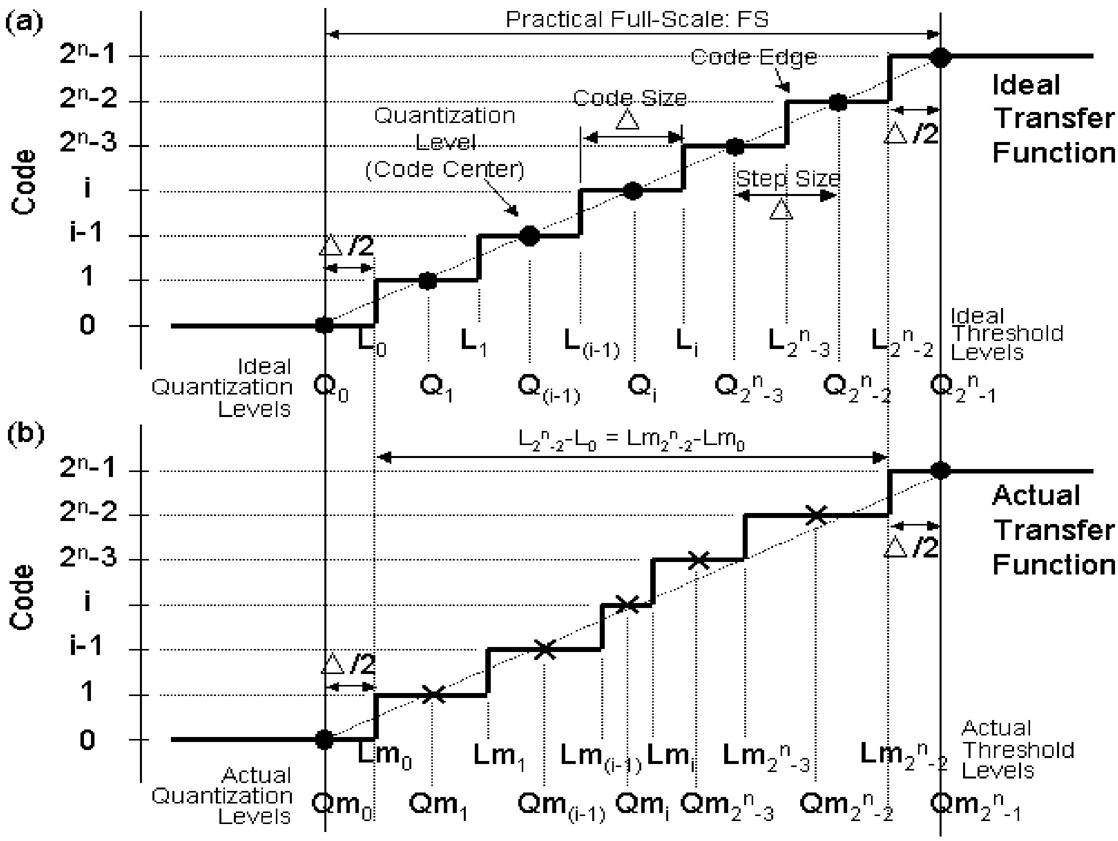

Figure 1 **n-bit Linear ADC Transfer Function**

The ideal quantization level points are located on the straight line. There are 2^n-1 quantization levels existing from code 0 to 2^n-1. Therefore the ideal step size Δ can be defined as below.

$$\Delta = \frac{Q_{2^n-1} - Q_0}{2^n - 1} = \frac{Qm_{2^n-1} - Qm_0}{2^n - 1} \tag{5}$$

Then the terminal quantization level points Q_0 and Q_{2^n-1} are defined as below.

$$Q_0 = L_o - \frac{1}{2}\Delta \tag{6}$$

$$Q_{2^n-1} = L_{2^n-2} + \frac{1}{2}\Delta \tag{7}$$

Therefore practically actual threshold levels Lm_0 and Lm_{2^n-2} are identical to the ideal threshold levels L_0 and L_{2^n-2} respectively as below.

$$Lm_0 = L_0 \tag{8}$$

$$Lm_{2^n-2} = L_{2^n-2} \tag{9}$$

Using these terminal threshold levels, the ideal code size Δ can be described as below.

$$\Delta = \frac{L_{2^n-2} - L_o}{2^n - 2} = \frac{Lm_{2^n-2} - Lm_0}{2^n - 2} \tag{10}$$

Consequently all quantization and threshold levels can be described as below.

$$Q_i = Q_0 + i \cdot \Delta \qquad (i=0, 1, 2, ..., 2^n-1) \tag{11}$$

$$L_i = L_0 + i \cdot \Delta \qquad (i=0, 1, 2, ..., 2^n-2) \tag{12}$$

Linearity Definitions

Linearity is the most important performance index of ADC. The definition is found in JEDEC Standard JESD 99-1 [1] and IEEE Std. 1241-2000 [2] which is based on threshold levels. The standard defines end-point linearity errors based on code size and threshold level errors as Figure 2, where code size error (*dle*[*i*]) and linearity error (*ile*[*i*]) are shown. Conventionally we call the code size error as *differential linearity error* (DLE) or *differential nonlinearity* (DNL), and the linearity error as *integral linearity error* (ILE) or *integral nonlinearity* (INL). Therefore DLE and ILE are used in this article.

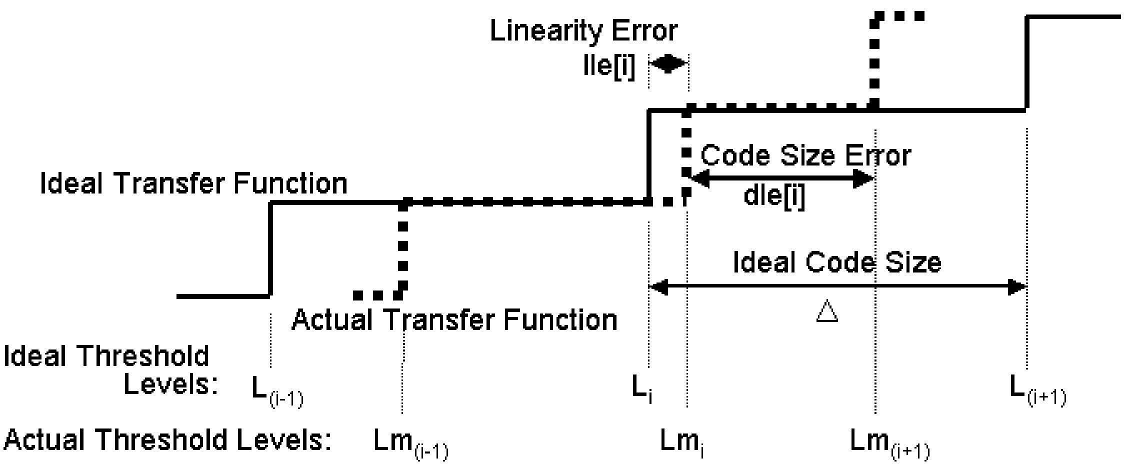

Figure 2 DLE and ILE

They are defined as below. The unit is LSB or least significant bit.

$$DLE[i] = \frac{Lm_{i+1} - Lm_i}{\Delta} - 1 \quad (13)$$

$$ILE[i] = \frac{Lm_i - L_i}{\Delta} \quad (14)$$

Ramp Histogram

Introduction

Ramp histogram is the simplest and the most straightforward test method of ADC linearity depicted in Figure 3. The stimulus signal should be a very linear ramp waveform. It must swing slightly larger than the ADC input range, otherwise the linearity cannot be tested correctly. It is very important point. Its slope must be slow enough for the ADC under test to generate multiple times every code. Since histogram method is a kind of statistical method, each code had better to occur many times, for example at least 10 counts. An image of code occurrence by a ramp is shown in Figure 3. Since the input ramp overloads to the input range of the DUT, code 0 and the full-scale code (2^n-1) occur extremely many times than the rest of the codes. Here *n* denotes the number of bits.

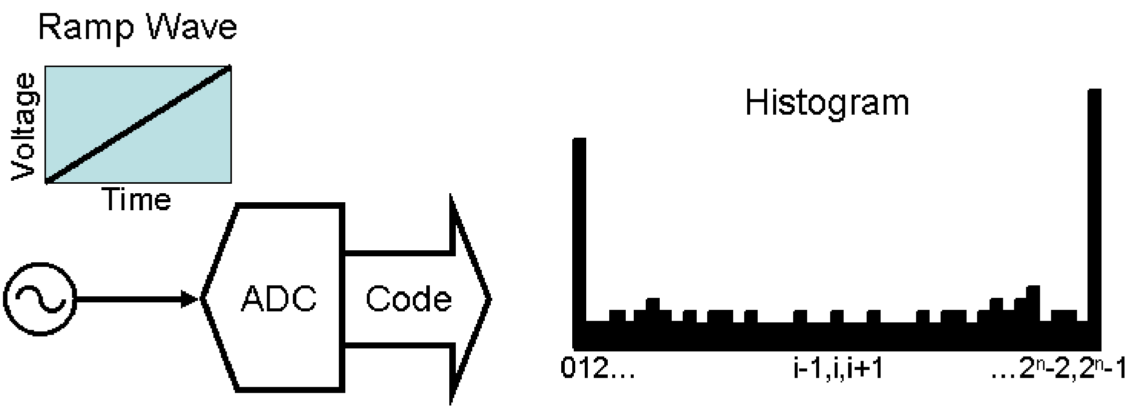

Figure 3 Ramp Histogram

Ramp Linearity Equation

A histogram by ramp signal looks as Figure 4.

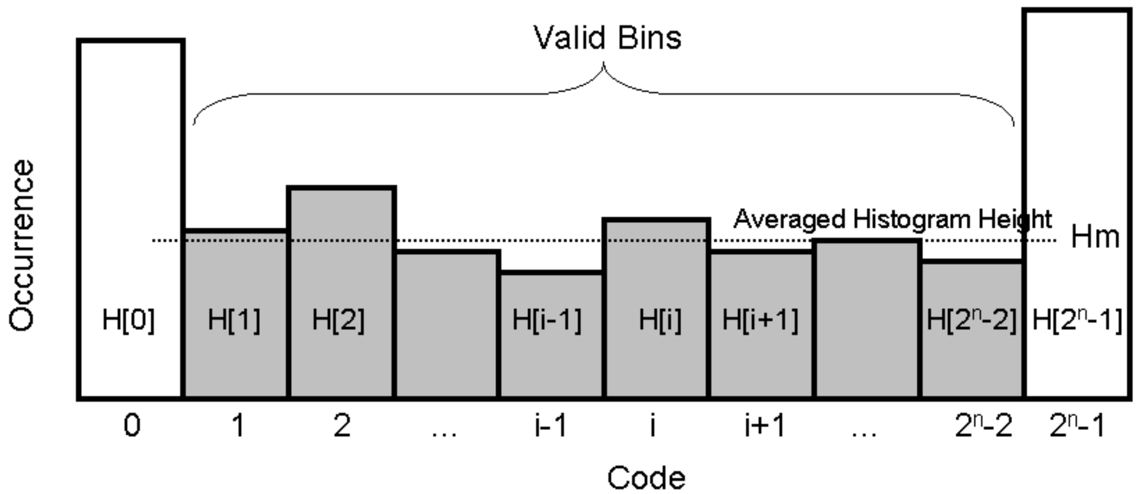

Figure 4 Ramp Histogram

The linearity is calculated as follows. Neglecting code 0 and code (2^n-1) counts, all the counts from code 1 through (2^n-2) are summed up. The average height (*Hm*) of the histogram from 1 through (2^n-2) is calculated as below.

$$Hm = \frac{\sum_{i=1}^{2^n-2} H[i]}{2^n - 2} \qquad (15)$$

A bin height is proportional to its code size. The average bin height *Hm* corresponds to the ideal code size so that *Hm* is the reference of each bin. Differential linearity error *DLE*[*i*] defined as Equation (13) is described using Equation (15) as below.

$$DLE[i] = \frac{H[i] - Hm}{Hm} = \frac{H[i]}{Hm} - 1 \text{ [LSB]} \qquad (16)$$

where i=1,2,3, … ,2^n-2, and $DLE[0]=DLE[2^n-1]=0$ perfunctory.

Integral linearity error *ILE*[*i*] in Equation (14) is now modified as below.

$$ILE[i] = \sum_{k=1}^{i} DLE[k] \qquad \text{[LSB]} \qquad (17)$$

where $Lm_0=L_0$, and $ILE[0]=ILE[2^n-1]=0$ perfunctory.

Equation (17) shows that *ILE*[*i*] is derived as accumulation of *DLE*[*i*]. Step-by-step deriving procedure is described in Appendix section. Equations (16) and (17) are the linearity equations by using a ramp stimulus.

Sine Histogram

Introduction

Ramp wave method is quite simple in terms of linearity calculation because ramp histogram appears a flat linear profile. However, actually it is not so easy to generate a good linear ramp waveform. A precision active integrator circuit with a low-loss and low dielectric absorption capacitor is required to generate a good ramp. In regular IC tests, a high precision D/A converter is often utilized to generate a pseudo precision ramp waveform instead of an integrator.

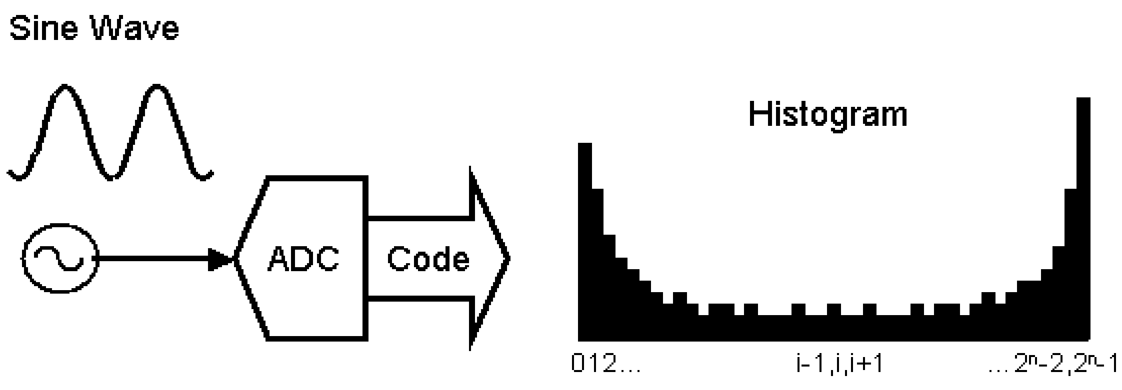

Figure 5 Sine Histogram

On the other hand, in sine wave histogram method, a very low distortion sine wave is required. It is relatively easy to generate such a low distortion sine wave because an appropriate low pass filter can easily remove distortions. However, since an ADC generates a non-flat histogram distribution depicted as Figure 5 for a sine wave, post processing of the sine histogram for linearity calculation becomes much more complex than the case of ramp histogram.

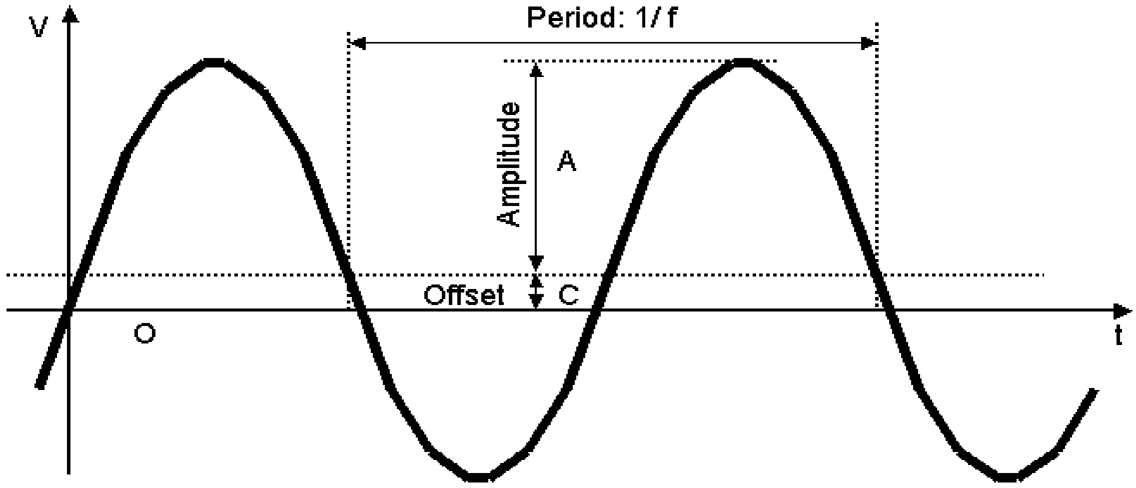

Figure 6 Sine Wave

Probability Density Function

A sine wave in Figure 6 can be expressed as follows;

$$V(t) = A\sin(2\pi ft + B) + C \tag{18}$$

where *A*, *B* and *C* are amplitude, phase and offset of the signal respectively, *f* is a frequency of the signal, and *t* shows time. (Phase offset *B* is not shown in the figure.)

Equation (18) is modified with regard to time *t* as below.

$$t = \frac{1}{2\pi f}\left\{\sin^{-1}\left(\frac{V(t) - C}{A}\right) - B\right\} \tag{19}$$

Histogram test is a statistical method. Histogram represents a probability of instantaneous voltage of the sine wave. A single period of the sine wave is a reciprocal of the frequency *f*. During the period of (1/*f*) the signal travels from (–*A*+*C*) to (*A*+*C*). When time is t_1 and t_2, the instantaneous voltage is located at the levels of V_1 and V_2 respectively as shown in Figure 7.

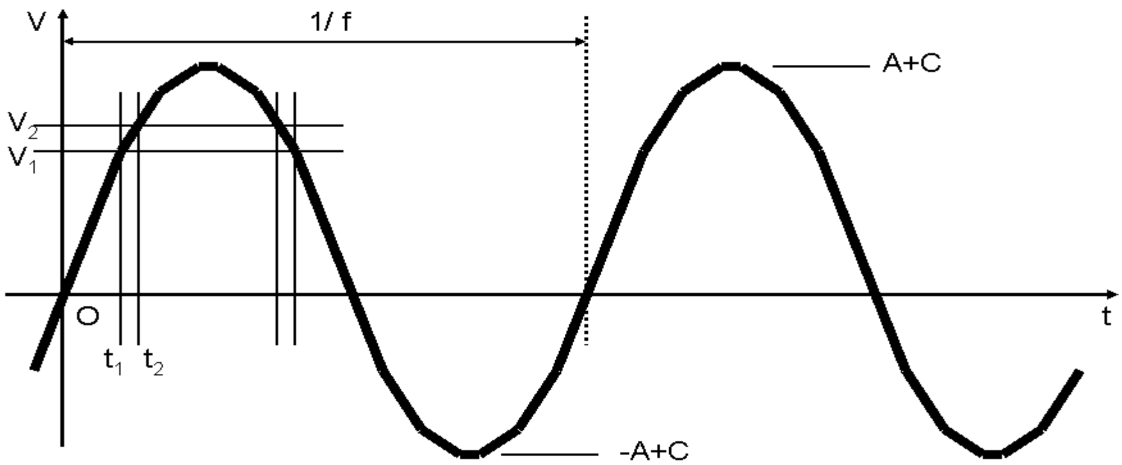

Figure 7 Probability between V_1 and V_2

Considering the probability that the signal exists between voltage V_1 and V_2, it occurs twice during the period of (1/*f*) as shown in Figure 7. Consequently probability density *P* that the signal exists between voltages V_1 and V_2 can be expressed as below.

$$P = \frac{2(t_2 - t_1)}{\left(\frac{1}{f}\right)} = \frac{1}{\pi}\left\{\sin^{-1}\left(\frac{V_2 - C}{A}\right) - \sin^{-1}\left(\frac{V_1 - C}{A}\right)\right\} \qquad (20)$$

This probability density looks a bathtub curve as Figure 8.

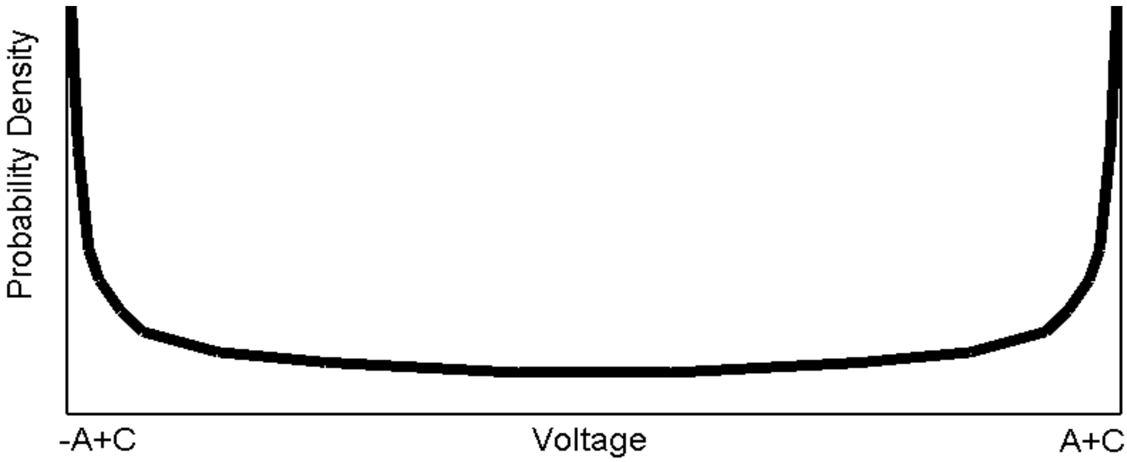

Figure 8 Probability Density

ADC's input range is *FS* in Figure 1. Figure 9 explains testing condition. Figure 9 (b) describes a probability density of the wave in (a). When analyzing linearity of an ADC, all available codes of ADC must be stimulated so that the test signal sine wave must be overloaded to the input range of the ADC. This is a very important point. So the conditions of $2A>FS$, $(A+C)>FS/2$ and $(-A+C)<(-FS/2)$ must be required. The points of +/-(FS/2) are identical to Q_{2^n-1} and Q_0 of the device.

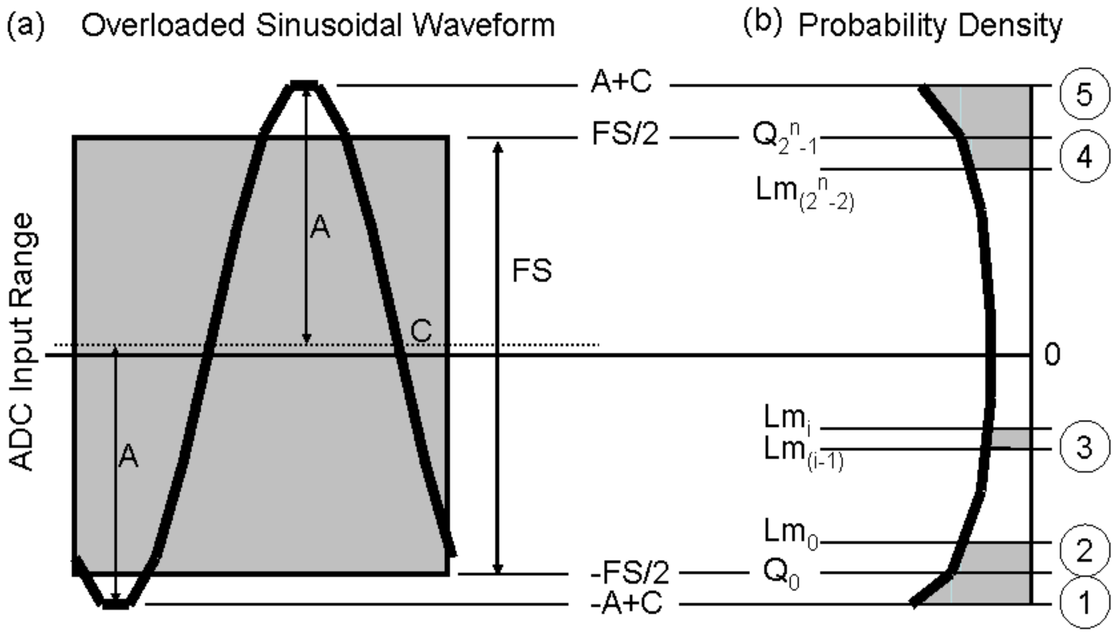

Figure 9 Sine Wave and Probability Density

When an instantaneous level of the signal is located at less than Lm_0, ADC generates code 0. When $P[0]$ denotes probability density that the code 0 occurs, $P[0]$ is the sum of areas numbered 1 and 2 in Figure 9 (b) so that $P[0]$ can be derived as below.

$$P[0] = \frac{1}{\pi}\sin^{-1}\left(\frac{Lm_0 - C}{A}\right) + \frac{1}{2} \qquad (21)$$

When the signal exists between levels Lm_{i-1} and Lm_i, ADC generate code *i*. Let *P*[*i*] denote the probability density that code *i* occurs. *P*[*i*] corresponds to the area numbered 3 in Figure 9 (b). *P*[*i*] can be described as below.

$$P[i] = \frac{1}{\pi}\left\{\sin^{-1}\left(\frac{Lm_i - C}{A}\right) - \sin^{-1}\left(\frac{Lm_{i-1} - C}{A}\right)\right\} \quad (i=1,2,...,2n-2) \qquad (22)$$

When the signal crosses over the threshold level Lm_{2^n-2}, ADC generates code (2^n-1). The probability density of code (2^n-1) is expressed as *P*[2^n-1] which is the sum of areas numbered 4 and 5 in Figure 9 (b). Then *P*[2^n-1] is described as below.

$$P[2^n - 1] = -\frac{1}{\pi}\sin^{-1}\left(\frac{Lm_{2^n-2} - C}{A}\right) + \frac{1}{2} \qquad (23)$$

In general, for calculating linearity of ADC, the measured histogram is directly compared to the ideal bathtub curve of the probability density Equations (21), (22) and (23). These equations contain the factors of amplitude *A* and offset *C,* which must be evaluated somehow in the calculation procedure. After constructing a sine histogram, firstly with analyzing the unbalance of the minimum and the maximum code bins, the offset and the amplitude of the sine wave should be estimated respectively, and then the histogram is compensated in terms of the offset and amplitude. [3][4][5] This procedure is really complex and induces calculation errors. More calculation steps make an impact to test throughput. This was a disadvantage in sine histogram method.

Cumulative Distribution Function

Instead of directly analyzing a probability density curve, another approach is to use a cumulative distribution function, which is an integral of probability density as shown in Figure 10.

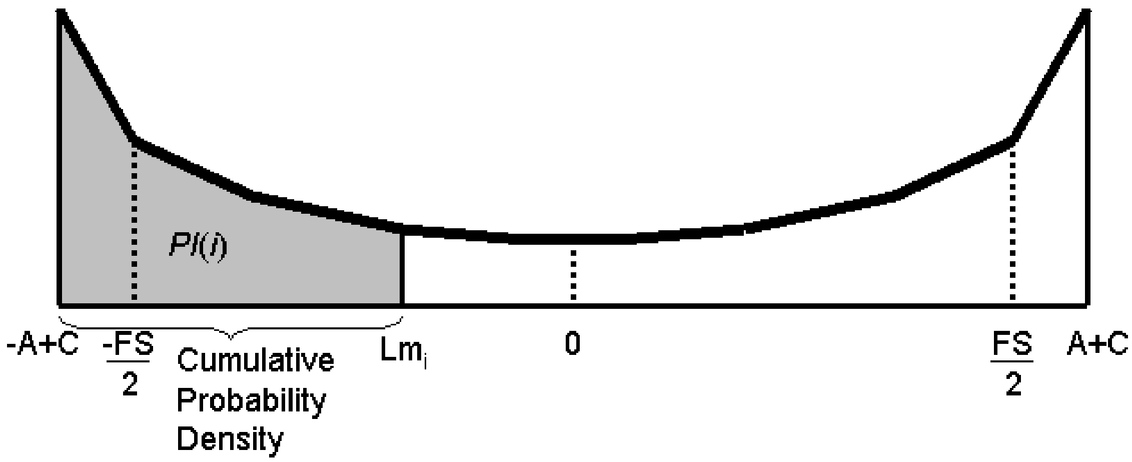

Figure 10 Cumulative Distribution from Code 0 to i

Let's integrate the bathtub curve. When *PI*[*i*] denotes the cumulative distribution that code from 0 to *i* occurs, with using Equations (21), (22) and (23), *PI*[*i*] is derived as follows.

$$PI[0] = P[0] \qquad (24)$$

$$PI[i] = \frac{1}{2} + \frac{1}{\pi}\sin^{-1}\left(\frac{Lm_i - C}{A}\right) \qquad (25)$$

$$PI[2^n - 1] = 1 \qquad (26)$$

By integrating the bathtub curve from code 0 to *i*, the cumulative distribution function looks as Figure 11.

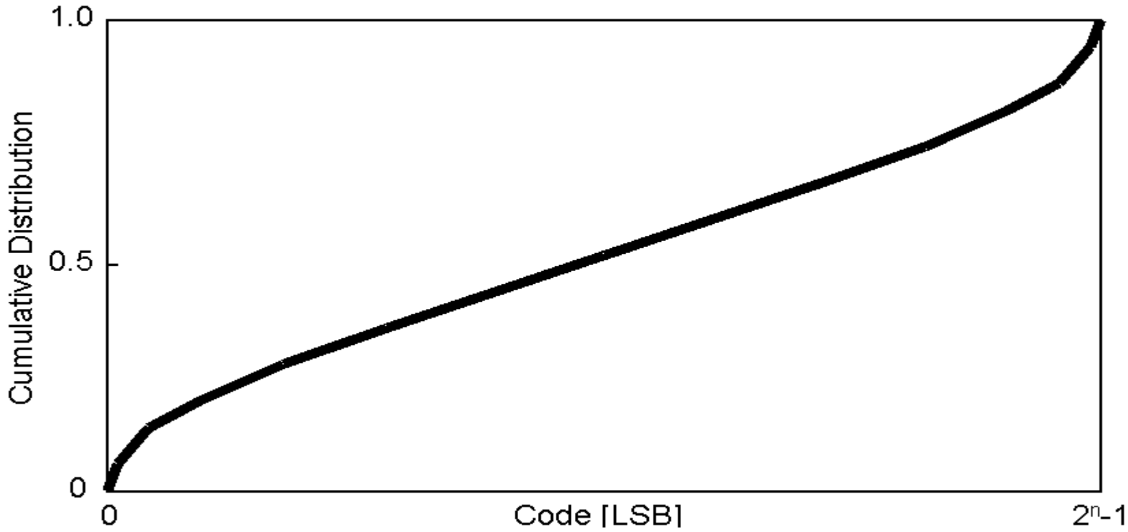

Figure 11 Cumulative Distribution Function

With modifying Equation (25), a threshold level Lm_i can be expressed with regard to *PI*[*i*] as below.

$$Lm_i = A\sin\left(\pi PI[i] - \frac{\pi}{2}\right) + C = C - A\cos\left(\pi PI[i]\right) \qquad (27)$$

Linearity Equations

By applying Equation (27) into Equations (13) and (14) with considering Equations (1) to (12), the differential linearity error DLE[i] and the integral linearity error ILE[i] are derived as follows.

$$DLE[i] = \left(2^n - 2\right)\frac{\cos\left(\pi PI[i]\right) - \cos\left(\pi PI[i+1]\right)}{\cos\left(\pi PI[0]\right) - \cos\left(\pi PI[2^n - 2]\right)} - 1$$

$(i=0,1,2,\ldots,2^n\text{-}3)$ (28)

$$ILE[i] = \left(2^n - 2\right)\frac{\cos\left(\pi PI[0]\right) - \cos\left(\pi PI[i]\right)}{\cos\left(\pi PI[0]\right) - \cos\left(\pi PI[2^n - 2]\right)} - i$$

$(i=0,1,2,\ldots,2^n\text{-}3)$ (29)

where $DLE[2^n\text{-}2]=ILE[2^n\text{-}2]=0$ perfunctory.

Advantage of the Equations

Equations (28) and (29) of *DLE*[*i*] and *ILE*[*i*] consist of cumulative probability *PI*[*i*] only, and they do not contain any parameters of *A*, *B* or *C* of the sine wave applied. This is the most important key of the equations. This means overload level and offset of input sine wave do not affect linearity error calculation. Consequently when applying these equations to sine histogram method, all you have to do is to ensure test signals overload the input range of ADC in order to stimulate all valid codes of the device, and you need not care about how much it overloads nor how much it offsets. This makes the test procedure very simple. Since you need not precisely adjust the test signal level and offset, no pre-test is required.

Equations (28) and (29) are functions of *PI*[*i*] only which can be calculated by the histogram straightforward. Therefore simple data processing without any compensation contributes higher throughput and precision. These are the most important advantage of the equations derived in this article.

Appendix

Precise deriving procedure of each equation is described in this section.

Equation (17)

$$
\begin{aligned}
ILE[i] &= \frac{Lm_i - L_i}{\Delta} = \frac{Lm_i - L_0 + i \cdot \Delta}{\Delta} \\
&= \frac{Lm_i - Lm_{i-1}}{\Delta} + \frac{Lm_{i-1} - Lm_{i-2}}{\Delta} + \ldots + \frac{Lm_2 - Lm_1}{\Delta} + \frac{Lm_1 - Lm_0}{\Delta} + \frac{Lm_0 - L_0}{\Delta} - i \\
&= \frac{Lm_i - Lm_{i-1}}{\Delta} + \frac{Lm_{i-1} - Lm_{i-2}}{\Delta} + \ldots + \frac{Lm_2 - Lm_1}{\Delta} + \frac{Lm_1 - Lm_0}{\Delta} - i \\
&= \sum_{k=1}^{i} \frac{Lm_k - Lm_{k-1}}{\Delta} - i = \sum_{k=1}^{i} \left(\frac{Lm_k - Lm_{k-1}}{\Delta} - 1 \right) \\
&= \sum_{k=1}^{i} DLE[k]
\end{aligned}
$$

Equation (21)

$$
\begin{aligned}
P[0] &= \frac{1}{\pi} \left\{ \sin^{-1}\left(\frac{\left(-\frac{FS}{2}\right) - C}{A} \right) - \sin^{-1}\left(\frac{(-A + C) - C}{A} \right) \right\} + \frac{1}{\pi} \left\{ \sin^{-1}\left(\frac{Lm_0 - C}{A} \right) - \sin^{-1}\left(\frac{\left(-\frac{FS}{2}\right) - C}{A} \right) \right\} \\
&= \frac{1}{\pi} \left\{ \sin^{-1}\left(\frac{Lm_0 - C}{A} \right) - \sin^{-1}(-1) \right\} \\
&= \frac{1}{\pi} \left\{ \sin^{-1}\left(\frac{Lm_0 - C}{A} \right) + \frac{\pi}{2} \right\} \\
&= \frac{1}{\pi} \sin^{-1}\left(\frac{Lm_0 - C}{A} \right) + \frac{1}{2}
\end{aligned}
$$

Equation (22)

$$P[2^n-1]=\frac{1}{\pi}\left\{\sin^{-1}\left(\frac{\frac{FS}{2}-C}{A}\right)-\sin^{-1}\left(\frac{Lm_{2^n-2}-C}{A}\right)\right\}+\frac{1}{\pi}\left\{\sin^{-1}\left(\frac{(A+C)-C}{A}\right)-\sin^{-1}\left(\frac{\frac{FS}{2}-C}{A}\right)\right\}$$

$$=\frac{1}{\pi}\left\{-\sin^{-1}\left(\frac{Lm_{2^n-2}-C}{A}\right)+\sin^{-1}(1)\right\}$$

$$=\frac{1}{\pi}\left\{-\sin^{-1}\left(\frac{Lm_{2^n-2}-C}{A}\right)+\frac{\pi}{2}\right\}$$

$$=\frac{1}{\pi}\sin^{-1}\left(\frac{Lm_{2^n-2}-C}{A}\right)+\frac{1}{2}$$

Equation (25)

$$PI[i]=\sum_{k=0}^{i}P[k]=P[0]+\sum_{k=1}^{i}P[k]$$

$$=\left\{\frac{1}{\pi}\sin^{-1}\left(\frac{Lm_0-C}{A}\right)+\frac{1}{2}\right\}+\sum_{k=1}^{i}\left(\frac{1}{\pi}\left\{\sin^{-1}\left(\frac{Lm_k-C}{A}\right)-\sin^{-1}\left(\frac{Lm_{k-1}-C}{A}\right)\right\}\right)$$

$$=\frac{1}{2}+\frac{1}{\pi}\left\{\sin^{-1}\left(\frac{Lm_0-C}{A}\right)+\sin^{-1}\left(\frac{Lm_1-C}{A}\right)-\sin^{-1}\left(\frac{Lm_0-C}{A}\right)+\ldots+\sin^{-1}\left(\frac{Lm_i-C}{A}\right)-\sin^{-1}\left(\frac{Lm_{i-1}-C}{A}\right)\right\}$$

$$=\frac{1}{2}+\frac{1}{\pi}\sin^{-1}\left(\frac{Lm_i-C}{A}\right)$$

Equation (26)

$$PI[2^n-1]=\sum_{k=0}^{2^n-1}P[k]$$

$$=PI[2^n-2]+P[2^n-1]$$

$$=\left\{\frac{1}{2}+\frac{1}{\pi}\sin^{-1}\left(\frac{Lm_{2^n-2}-C}{A}\right)\right\}+\left\{-\frac{1}{\pi}\sin^{-1}\left(\frac{Lm_{2^n-2}-C}{A}\right)+\frac{1}{2}\right\}$$

$$=\frac{1}{2}+\frac{1}{2}=1$$

Equation (27)

$$PI[i]=\frac{1}{2}+\frac{1}{\pi}\sin^{-1}\left(\frac{Lm_i-C}{A}\right)$$

$$\sin^{-1}\left(\frac{Lm_i-C}{A}\right)=\pi\left(PI[i]-\frac{1}{2}\right)$$

$$Lm_i=A\sin\left(\pi PI[i]-\frac{\pi}{2}\right)+C=C-A\cos(\pi PI[i])$$

Then Δ and Qm_i can be expressed with Lm_i as below.

$$\Delta = \frac{Lm_{2^n-2} - Lm_0}{2^n - 2}$$

$$= \frac{\{C - A\cos(\pi PI[2^n - 2])\} - \{C - A\cos(\pi PI[0])\}}{2^n - 2}$$

$$= \frac{A}{2^n - 2} \cdot \{\cos(\pi PI[0]) - \cos(\pi PI[2^n - 2])\}$$

$$Qm_i = \frac{Lm_{i-1} + Lm_i}{2}$$

$$= \frac{\{C - A\cos(\pi PI[i-1])\} - \{C - A\cos(\pi PI[i])\}}{2}$$

$$= C - \frac{A}{2}\{\cos(\pi PI[i-1]) + \cos(\pi PI[i])\}$$

Equation (28)

$$DLE[i] = \frac{Lm_{i+1} - Lm_i}{\Delta} - 1$$

$$= \frac{\{C - A\cos(\pi PI[i+1])\} - \{C - A\cos(\pi PI[i])\}}{\frac{A}{2^n - 2} \cdot \{\cos(\pi PI[0]) - \cos(\pi PI[2^n - 2])\}} - 1$$

$$= (2^{n-1} - 2) \cdot \frac{\cos(\pi PI[i]) - \cos(\pi PI[i+1])}{\cos(\pi PI[0]) - \cos(\pi PI[2^n - 2])} - 1$$

Equation (29)

$$ILE[i] = \frac{Lm_i - L_i}{\Delta} = \frac{Lm_i - (L_0 + \Delta \cdot i)}{\Delta} = \frac{Lm_i - L_0}{\Delta} + i = \frac{Lm_i - Lm_o}{\Delta} - i$$

$$= \frac{\{C - A\cos(\pi PI[i])\} - \{C - A\cos(\pi PI[0])\}}{\frac{A}{2^n - 2} \cdot \{\cos(\pi PI[0]) - \cos(\pi PI[2^n - 2])\}}$$

$$= (2^n - 2) \cdot \frac{\cos(\pi PI[0]) - \cos(\pi PI[i])}{\cos(\pi PI[0]) - \cos(\pi PI[2^n - 2])} - i$$

References

[1] "Terms, Definitions, and Letter Symbols for Analog-to-Digital and Digital-to-Analog Converters", JEDEC Standard No.99 Addendum No.1 (JESD99-1), *Electronic Industries Association*, 1989, p. 32.

[2] "IEEE Standard for Terminology and Test Methods for Analog-to-Digital Converters", IEEE Std 1241-2000, 2000, p.3

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[4] Mark Burns and Gordon W. Roberts, "An Introduction to Mixed-Signal SOC Test and Measurement", *Oxford University Press*, 2001, pp. 462-467

[5] Walt Kester, "Analog-Digital Conversion", *Analog Devices*, 2004, pp. 5.38-5.45