Hideo Okawara's
Mixed Signal Lecture Series

DSP-Based Testing – Fundamentals 27
Multi-tone Under-sampling Conditioning

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Preface to the Series

ADC and DAC are the most typical mixed signal devices. In mixed signal testing, analog stimulus signal is generated by an arbitrary waveform generator (AWG) which employs a D/A converter inside, and analog signal is measured by a digitizer or a sampler which employs an A/D converter inside. The stimulus signal is created with mathematical method, and the measured signal is processed with mathematical method, extracting various parameters. It is based on digital signal processing (DSP) so that our test methodologies are often called DSP-based testing.

Test/application engineers in the mixed signal field should have thorough knowledge about DSP-based testing. FFT (Fast Fourier Transform) is the most powerful tool here. This corner will deliver a series of fundamental knowledge of DSP-based testing, especially FFT and its related topics. It will help test/application engineers comprehend what the DSP-based testing is and assorted techniques.

Editor's Note

For other articles in this series, please visit the Verigy web site at www.verigy.com/go/gosemi.

Preface

Wireless communication devices are usually tested RF systems with frequency down-converters and IF digitizers. Recent wireless communication systems deploy various wideband signals such as OFDM, UWB whose spectrum is spread over widely. In these applications, the signal bandwidth is extremely wide, for instance, several hundreds MHz so that regular real-time digitizers cannot cover the entire bandwidth. In this situation, waveform samplers may be able to provide a solution with utilizing extremely wide analog input bandwidth and under-sampling technique. Even if a sampler has very wide input bandwidth, its actual baseband range is still limited as digitizers do. So when utilizing the sampler to capture wider signals than its Nyquist bandwidth, you have to carefully plan the measurement condition not to conflict the aliasing frequencies in the baseband. So the sampler conditioning for multi-tone signals is discussed in this issue.

Folded Frequency Domain Spectrum in Under-sampling

Waveform samplers have extremely wide analog input bandwidth, and perform under-sampling to capture much higher frequency signals than their actual sampling frequency. Figure 1 illustrates how samplers capture high frequency signals with under-sampling technique. The figure depicts a panoramic frequency spectrum domain. With defining the sampling frequency and the number of points as *Fs* and *N* respectively, the wide-spread frequency domain can virtually be divided into half-Fs or half-N size pages. In Figure 1, each one of the pages is distinguished by colors. The author calls bright colored pages as front pages and gray pages as back pages. When under-sampling is performed, the entire pages of the panoramic frequency domain are folded and degenerated into the baseband page. All the spectral information is aliased in the baseband by under-sampling. This is the concept that samplers measure extremely wide band signals.

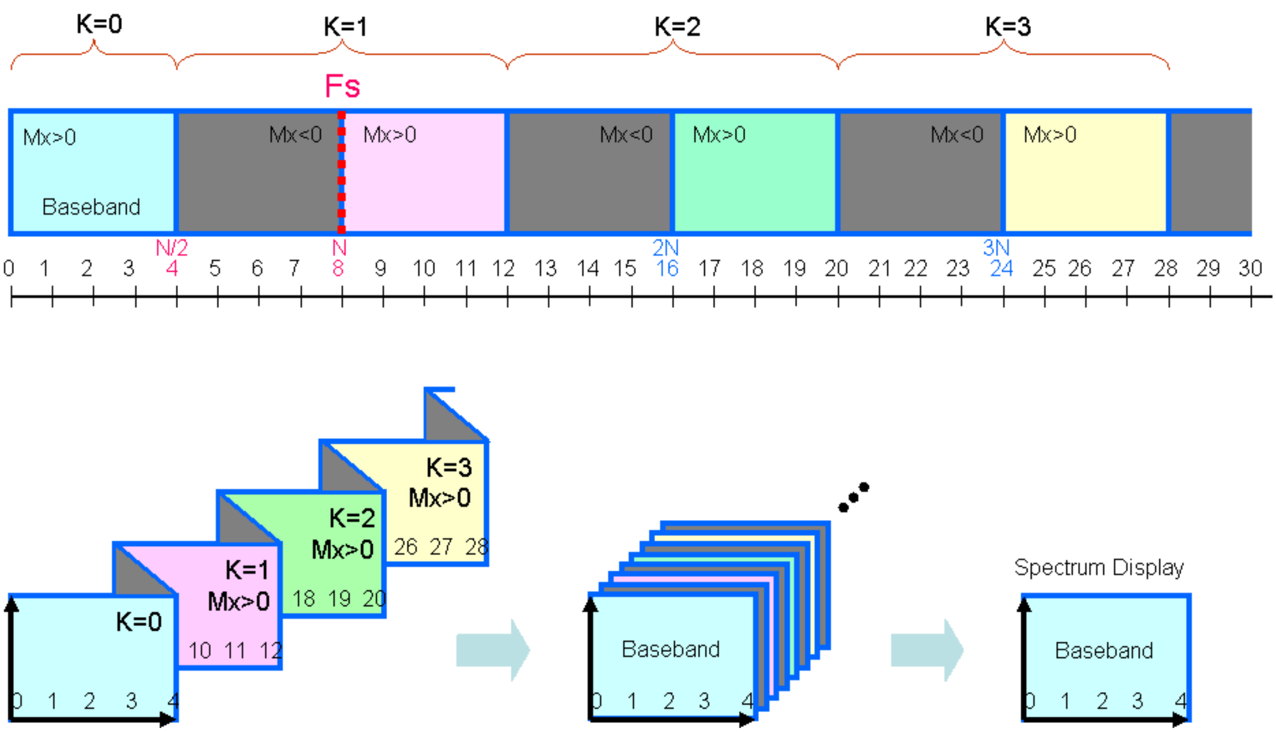

Figure 1: Page Concept of Under-sampling

The number of spectrum bins in the baseband is limited and a half of the number of the sampling points *N*. So you have to carefully settle the *Fs* and *N* for the entire aliased test signal tones not to conflict with each other on the baseband frequency axis.

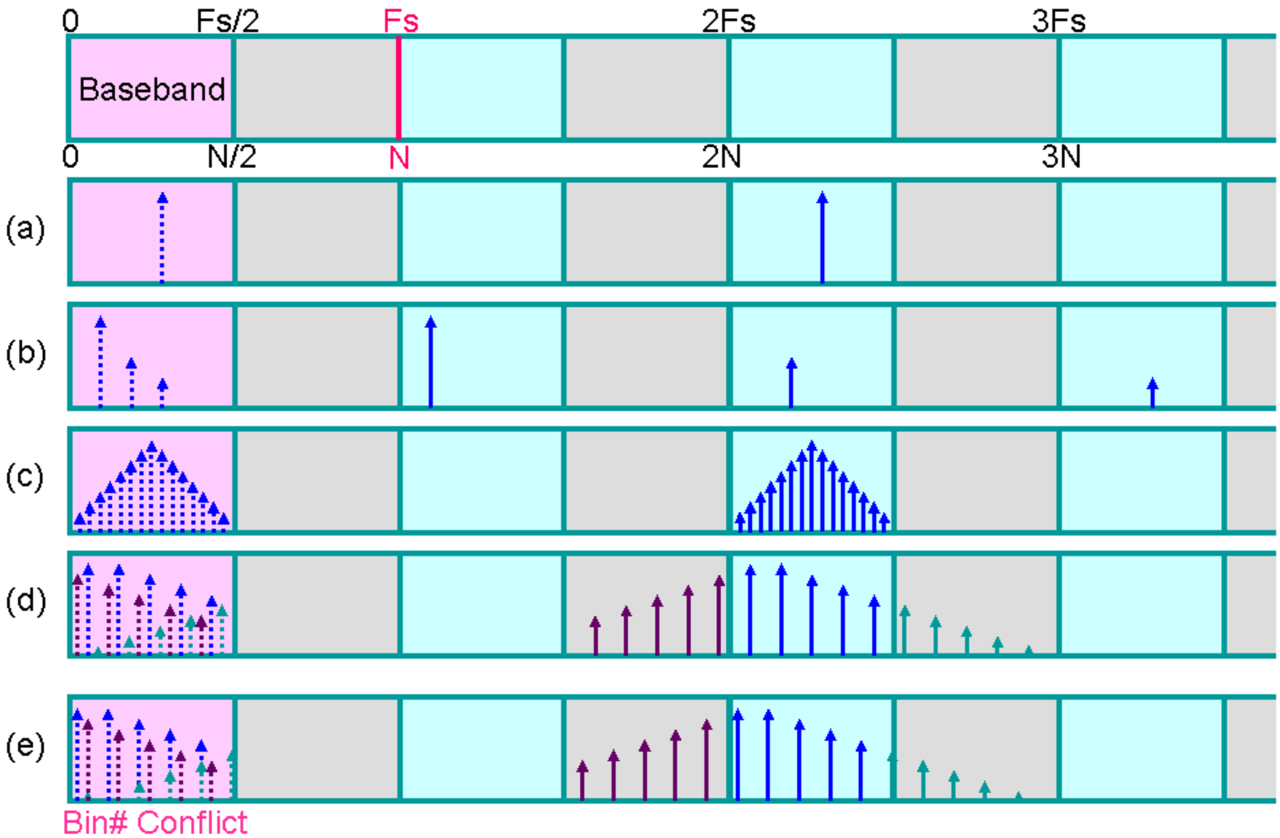

Figure 2: Measurable Signals

Figure 2 illustrates various situations of test signals. (a) shows a pure single tone which can be measured obviously without any difficult problem. (b) is a case that the fundamental tone and its 2nd and 3rd harmonics are distributed in the three different pages. They are aliased in the baseband with normal order in this picture with simple coherency conditioning. (c) shows a wideband signal which is spread within a single page so that the entire signal is aliased and exactly replicated in the baseband. (a) (b) and (c) are easy cases for successful under-sampling. (d) and (e) are extremely wideband signals which spread over across several pages. In this case the aliased spectral lines are aliased in the single baseband and wrapped over several times so that some of the lines may conflict with each other. (d) and (e) are very similar signals. In the case of (d) the signal is successfully folded back without aliased bin number duplication. On the other hand, in the case of (e) some of the lines get duplicated their aliased bin numbers. Once their bin numbers conflict, you cannot separate each one of them any more. So you must carefully make a test plan to avoid the situation as (e). For detail of the basic discussion about under-sampling, please see the archived newsletter articles.[1] The following sections are discussed about how to handle with this kind of multi-tone signals.

[1] Hideo Okawara's Mixed Signal Lecture Series "DSP Based Testing – Fundamentals 8 – Under-sampling" December 2008, "Fundamentals 9 – Under-sampling 2" January 2009, "Fundamentals 10 – Under-sampling 3" February 2009

Consideration of Frequency Resolution

Multi-tone test signals that ATE measures are usually allocated regularly on the frequency axis as shown in Figure 3. The *L* lines of spectra is described with the equal frequency spacing of $\triangle F$ in Figure 3. So the total bandwidth of the test signal is $L \cdot \triangle F$ in this case. Considering the entire spectral lines of the signal to be captured in the baseband of the sampler, the number of sampling points *N* must be greater than twice of the number of test signal lines because the number of bins in the baseband is N/2. So firstly, the equation of $N>2L$ is derived. Considering the aliasing or folding back mechanism, the baseband bin number of the sampler can be extended more than N/2 far away up to the full input bandwidth. In order to capture each one of the test signal bins, the original signal locations must exactly be matched to the extended bin numbers of the sampler so that the $\triangle F$ must be the multiple of the frequency resolution of the sampling condition. Consequently the second required condition is derived as $\triangle F = Nx \cdot Fres$, where *Nx* is an integer number, *Fres* is the frequency resolution of the sampler, and $Fres = Fs/N$.

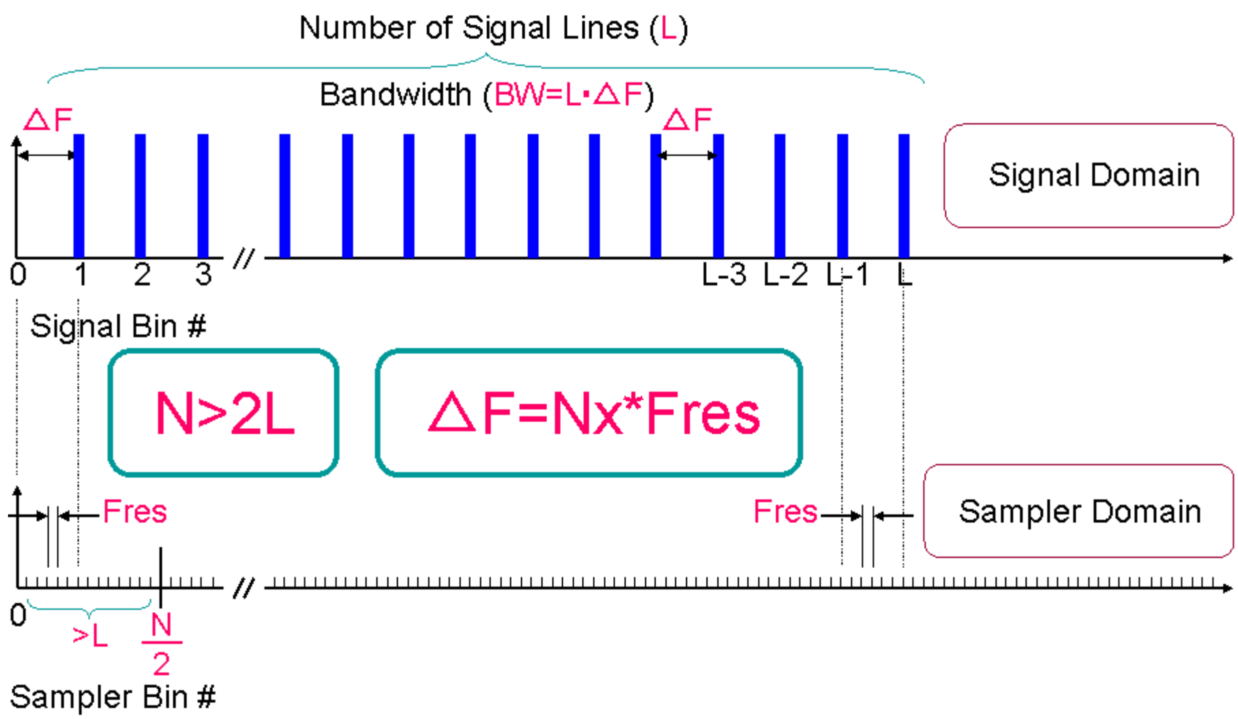

Figure 3: Multi-tone Frequency Resolution

Let's think of the parameter *Nx*. When a multi-tone signal spread over wider than $Fs/2$ is aliased in the baseband, the regular frequency locations are folded back many times. Figure 4 illustrates how the folding occurs. The original signal bin locations are depicted with green long tick lines and the sampler's bin locations are depicted blue short ticks. If *Nx* is an even number, when the axis is folded back, the green lines conflict each other as shown in (a). If *Nx* is an odd number, when the axis is folded back, the green lines get staggered and do not conflict as (b). So the parameter *Nx* should be an odd integer number.

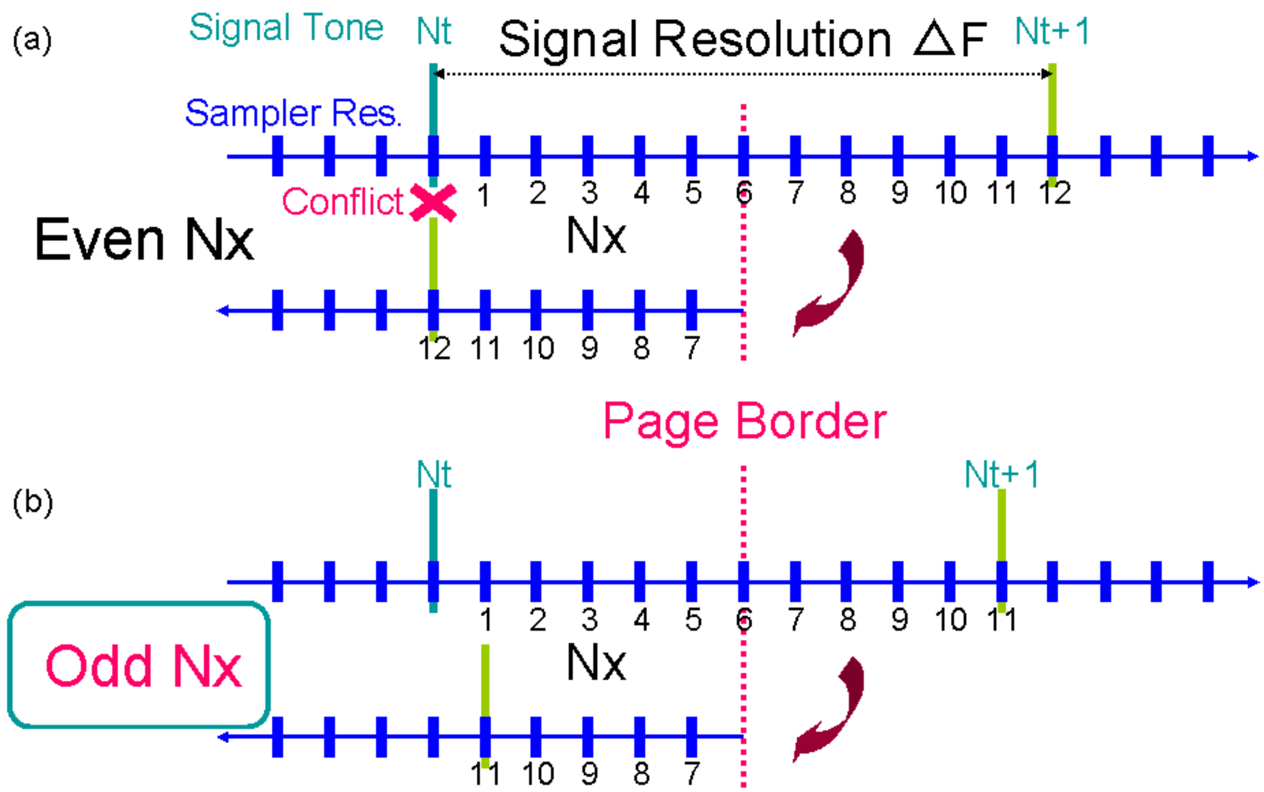

Figure 4: Nx Condition

Summary of Basic Conditions

With the discussions above, in order to make a successful under-sampling for wideband multi-tone signals, the basic conditions are derived as follows.

1. $N>2L$
2. $\triangle F=Nx*Fs/N$
3. Nx=Odd Integer

So the decision process of the sampler test condition would be as follows. Firstly, the number of points N is decided based on the number of the target test signal bins. N is usually settled as 2^n because of the convenience for FFT. $\triangle F$ is usually a given number if the signal is a specific modulation signal, or probably you can decide it by yourself if the signal is a stimulus for wideband frequency response test. The sampling frequency of the sampler has its maximum available rate. Fs should be settled as high as possible to make the baseband as wide as possible. Now that N, $\triangle F$ and approximate Fs are decided, an appropriate Nx can be derived as an odd number. This is the basic process of sampling condition.

Let's see some concrete examples to comprehend more. The following examples are actually simulations of under-sampling.

Example 1: Filter Frequency Response Measurement (Simulation)

The first example is a frequency response test of a low pass filter whose cut of frequency is approximately 500MHz. Figure 5 illustrates the stimulus plan.

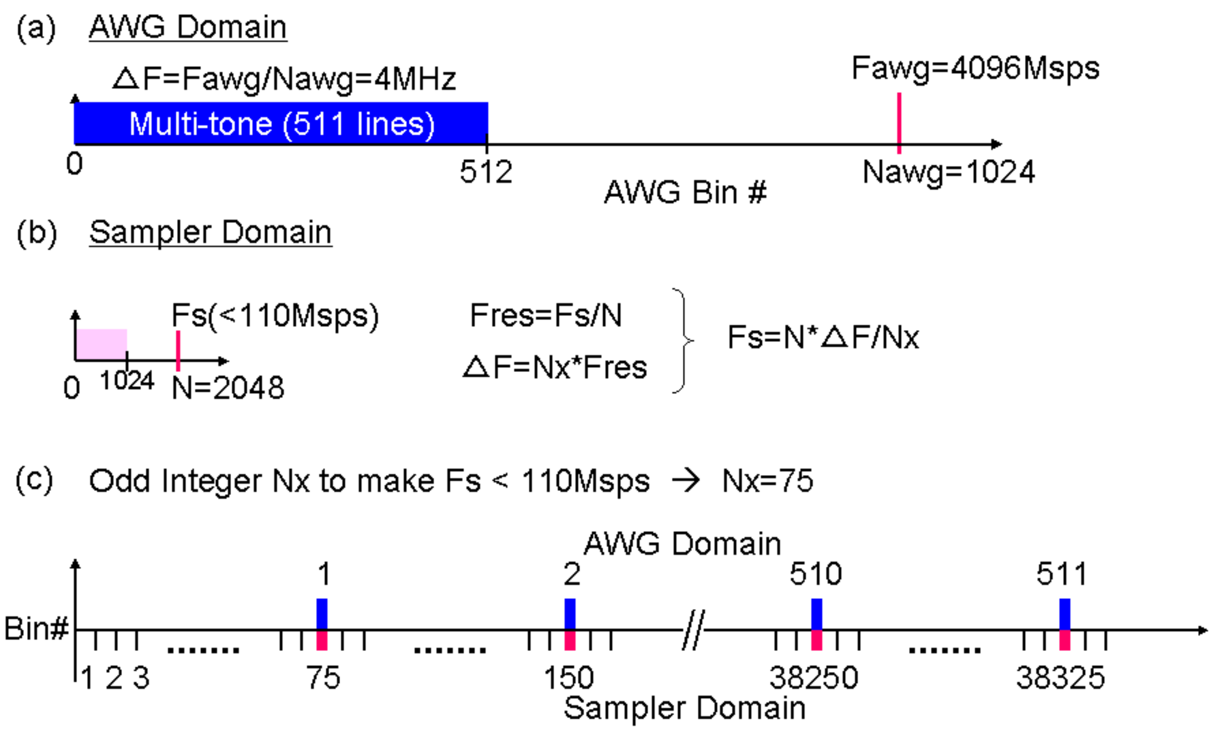

Figure 5: LPF Test Signal Plan

The stimulus signal is 511-line flat level multi-tone with a frequency spacing of 4MHz ($\triangle F$). So the signal is constructed by the tones of 4MHz, 8MHz, 12MHz, ... , 2044MHz. Since L=511, the number of sampling points N should be greater than 1022. So N is decided as 2048 here. Let's use the waveform sampler whose maximum sampling rate is 110Msps. Then the smallest possible Nx becomes 75, and consequently Fs is settled as 109.22666...Msps.

According to these conditions, the sampler captures the waveform shown in Figure 6. Its FFT spectrum is shown in the same figure as well. The raw waveform and its spectrum are scrambled so that they do not show any informative image at all. However, you can reconstruct the original waveform and its frequency spectrum with performing the waveform reshuffling DSP API. Using DSP_SHUFFLE() with the key number Nx, the waveform in Figure 6 is organized as the top picture of Figure 7 so that its FFT generates the informative frequency spectrum shown in the bottom picture in Figure 7. The spectrum clearly shows the LPF frequency response. Now that the number of points is 2048 and the spectral line spacing is 4MHz, the equivalent sampling rate becomes 8.192Gsps. This is the art of the under-sampling.

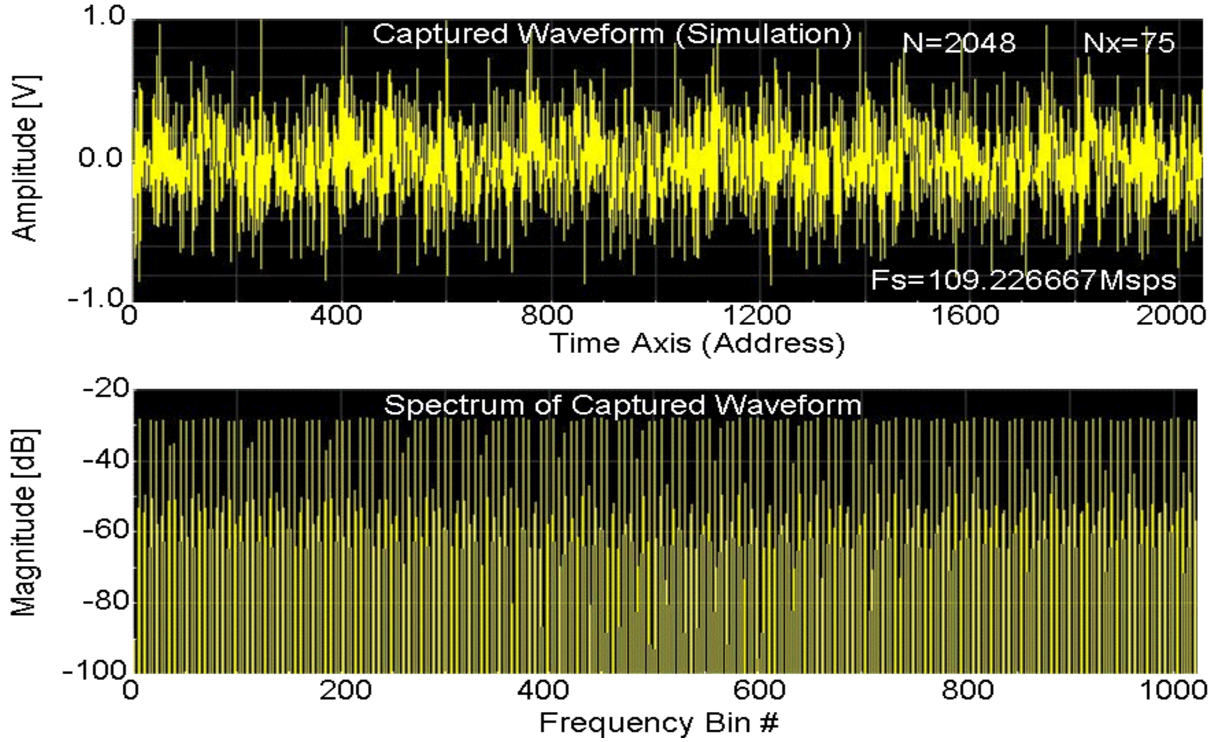

Figure 6: Captured Waveform and FFT Spectrum

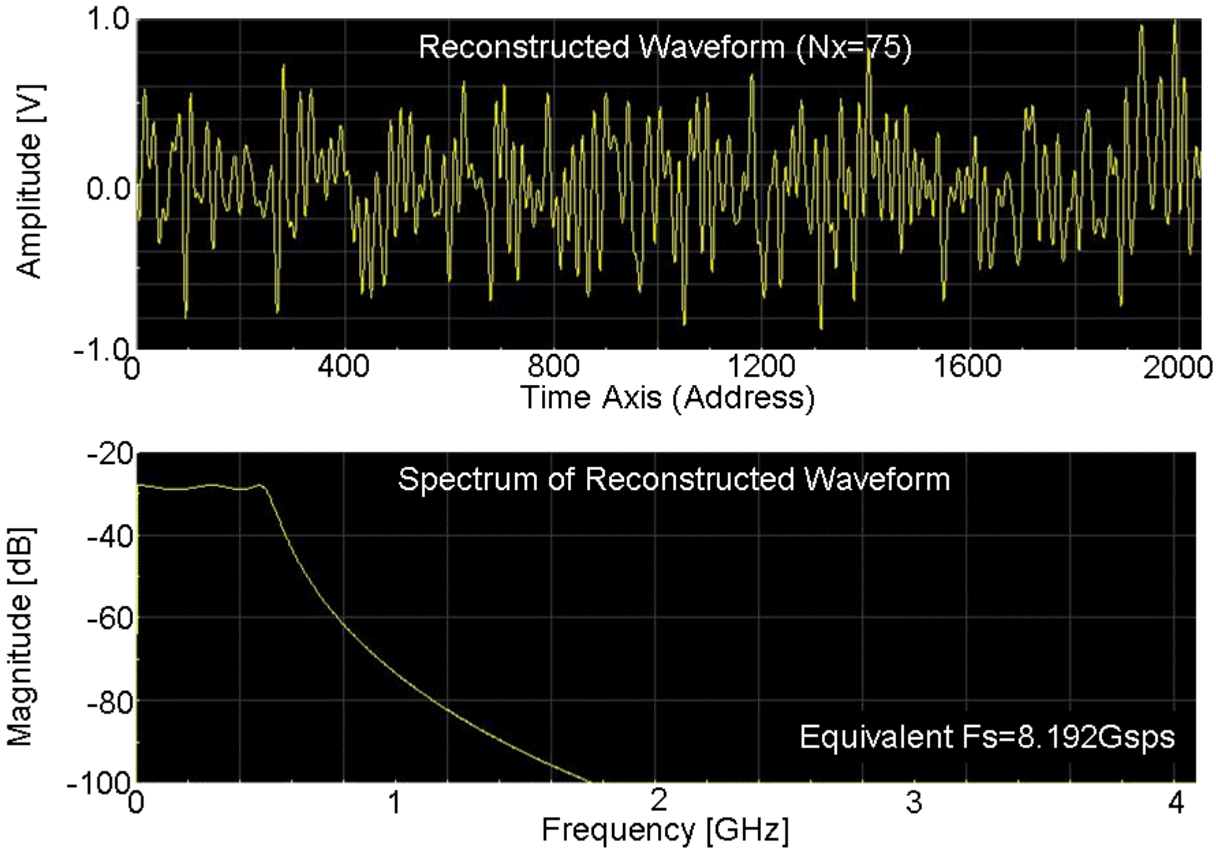

Figure 7: Reconstructed Waveform and FFT Spectrum

Example 2: High-speed Digital Bit Stream Measurement (Simulation)

Figure 8 shows 512 bits PRBS bit stream at 2048Mbps. This signal has a frequency spectrum as shown in Figure 9.

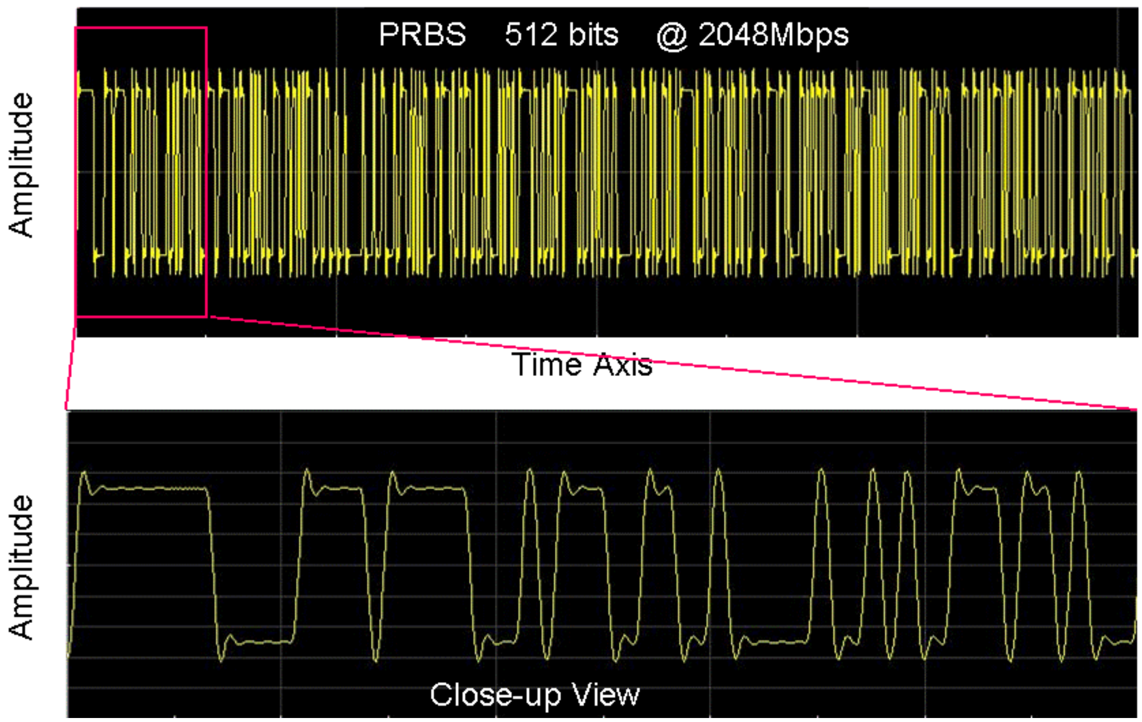

Figure 8:   PRBS Digital Bit Stream (2048Mbps 512-bits Looping)

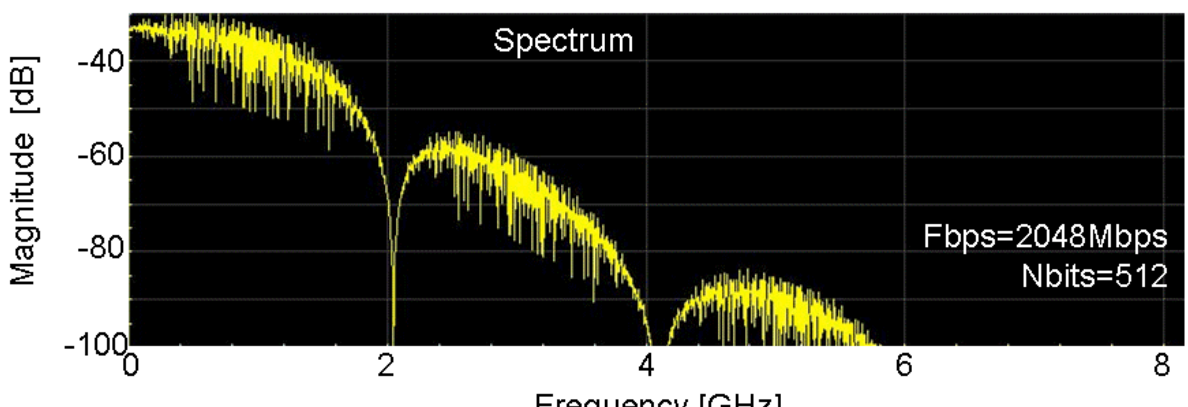

Figure 9:   Frequency Spectrum of PRBS

Since this PRBS is constructed with 512 bits of digital data, the main lobe and each side lobe is constructed with 512 lines of spectrum respectively. If the signal up to 8192MHz should be captured, the number of the target signal bins is 512x4=2048 which is *L*. The first condition is $N>2L$ so that *N* is decided as 8192 here. The bin spacing is 2048MHz/512=4MHz which is $\triangle F$. When the maximum sampling rate of the sampler is 110Msps, the appropriate *Nx* is settled as 299, and then the actual sampling rate becomes 109.591973244...Msps.

Using the condition described above, the captured waveform and its FFT spectrum are shown in Figure 10, which is not informative at all. Performing DSP_SHUFFLE() with the key number Nx (299), the waveform is organized as the top picture in Figure 11. The 512 bits of PRBS waveform is clearly reconstructed, and its FFT spectrum shows the similar spectrum to Figure 9.

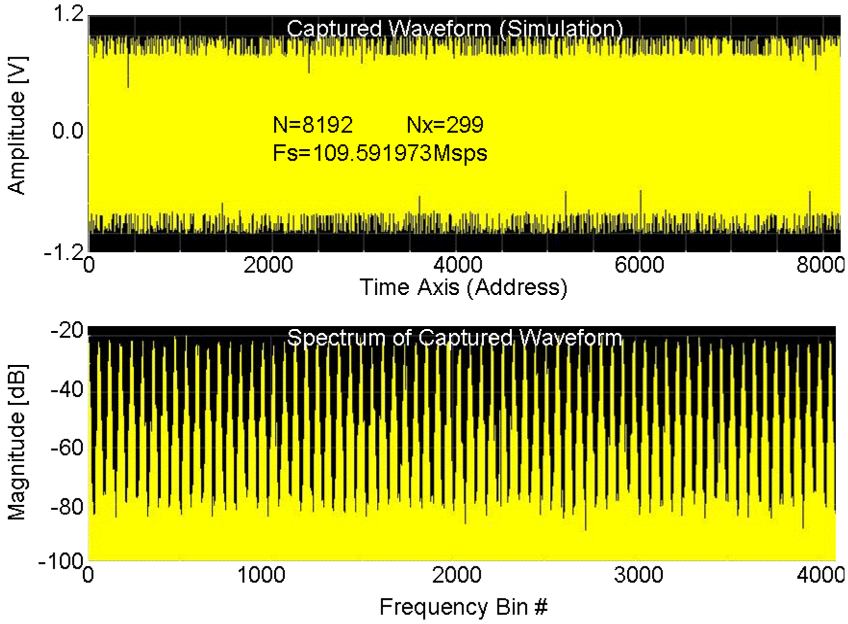

Figure 10: Capture Waveform and FFT Spectrum

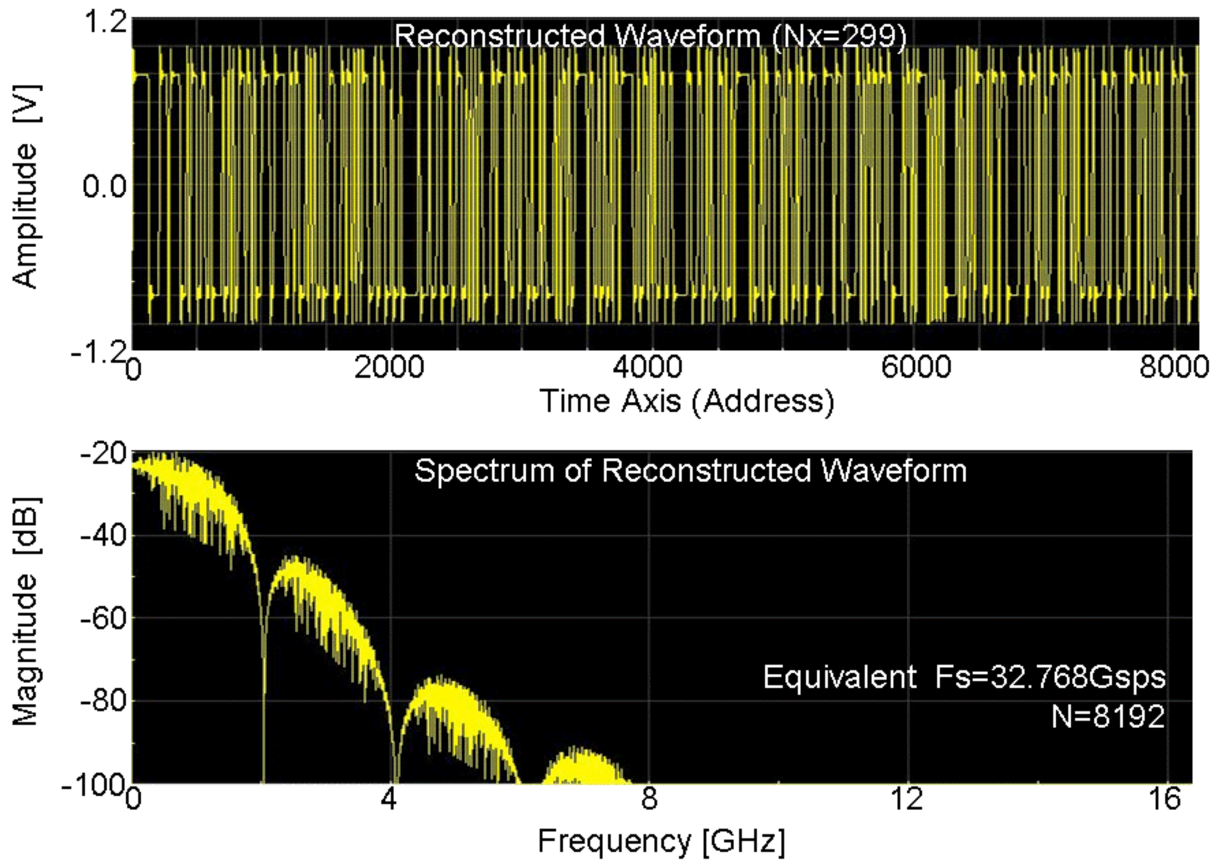

Figure 11: Reconstructed Waveform and FFT Spectrum (Nx=299)

The reconstructed bit stream in Figure 11 is sliced every two bits and overlaid in a single picture. Then the eye diagram is generated as shown in Figure 12.

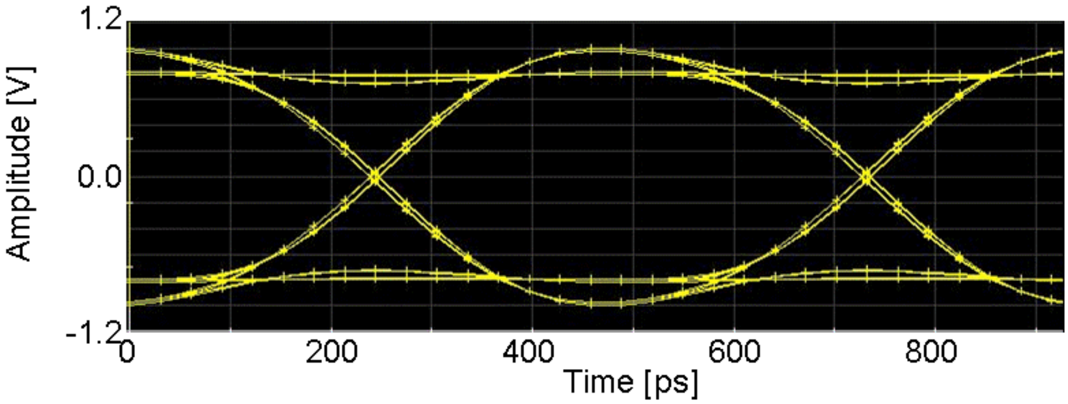

Figure 12: Eye Diagram Generated by Reconstructed Waveform

There are 32 time-locations marked in the Eye locus. They are the sampling points. The 8192 points construct the 512 bits so that 8192/512x2=32 specific locations make up the eye. Each one of the points is connected with lines.

Example 3: Localized Wideband Signal Measurement (Simulation)

This is an interesting measurement. Wireless communication systems employ wideband modulation signals such as CDMA, OFDM, UWB and so forth. These signals usually have very wideband but are localized in a certain bandwidth.

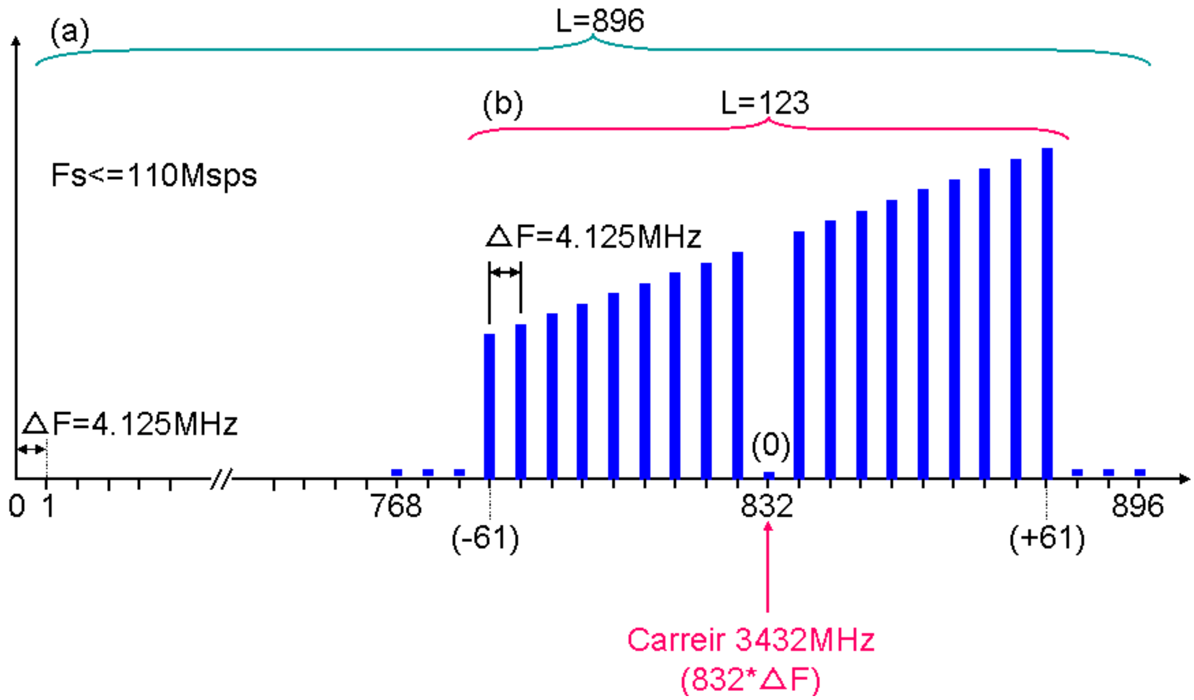

Figure 13: Localized Multi-tone Test Signal

Figure 13 illustrates a model signal emulating the carrier 3432MHz modulated by +/-64 lines of lower sideband and higher sideband respectively. The line spacing is 4.125MHz which is $\triangle F$. The carrier is located at the bin number of 832. The locations +/-62 to 64 are not used actually. So there are 122 tones besides the carrier. Consequently total 123 tones are arranged. In order to easily distinguish each one of the tones, the amplitude of the tones is intentionally modified as illustrated. The phase of each tone is just randomized.

The straightforward plan to capture this signal is to cover the entire signal locations marked as (a) in Figure 13. Then the number of target locations is from bin 1 through 896 so that L=896. Then N (>2L) can be decided as 2048. $\triangle F$=4.125MHz and Fs<110Msps so that the appropriate Nx becomes 77 and the true Fs becomes 109.714285714...MHz.

Applying the conditions above, the waveform captured as the Figure 14 top picture and its FFT spectrum appears at the bottom in the figure. However they are not informative at all. So the waveform in Figure 14 is processed with DSP_SHUFFLE() with the key number Nx=77. Then the waveform is organized and the FFT spectrum appears as Figure 15. The spectrum is exactly localized around the bin number 832 with slant level response so that the signal in Figure 14 is successfully reconstructed in Figure 15. Since the original spectrum structure is reconstructed, the equivalent sampling rate becomes 8448Msps (=$N \cdot \triangle F$).

In Figure 15 bottom, you can find parameters N, M, K and Mx, which are based on the extended coherency equation, where Ft and Fs are the test signal frequency and the sampling frequency. M and N are the number of test signal cycles and the number of sampling points in the unit test period. Fs is actually an equivalent sampling rate in this case.

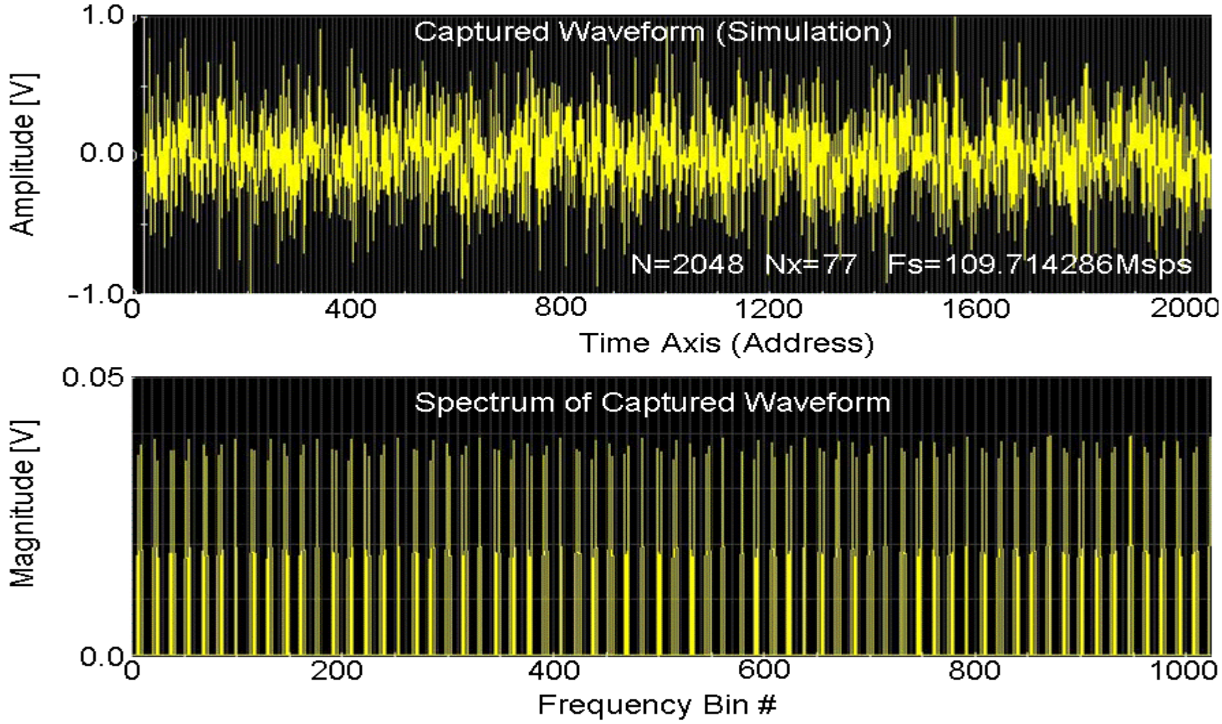

Figure 14: Captured Waveform and FFT Spectrum

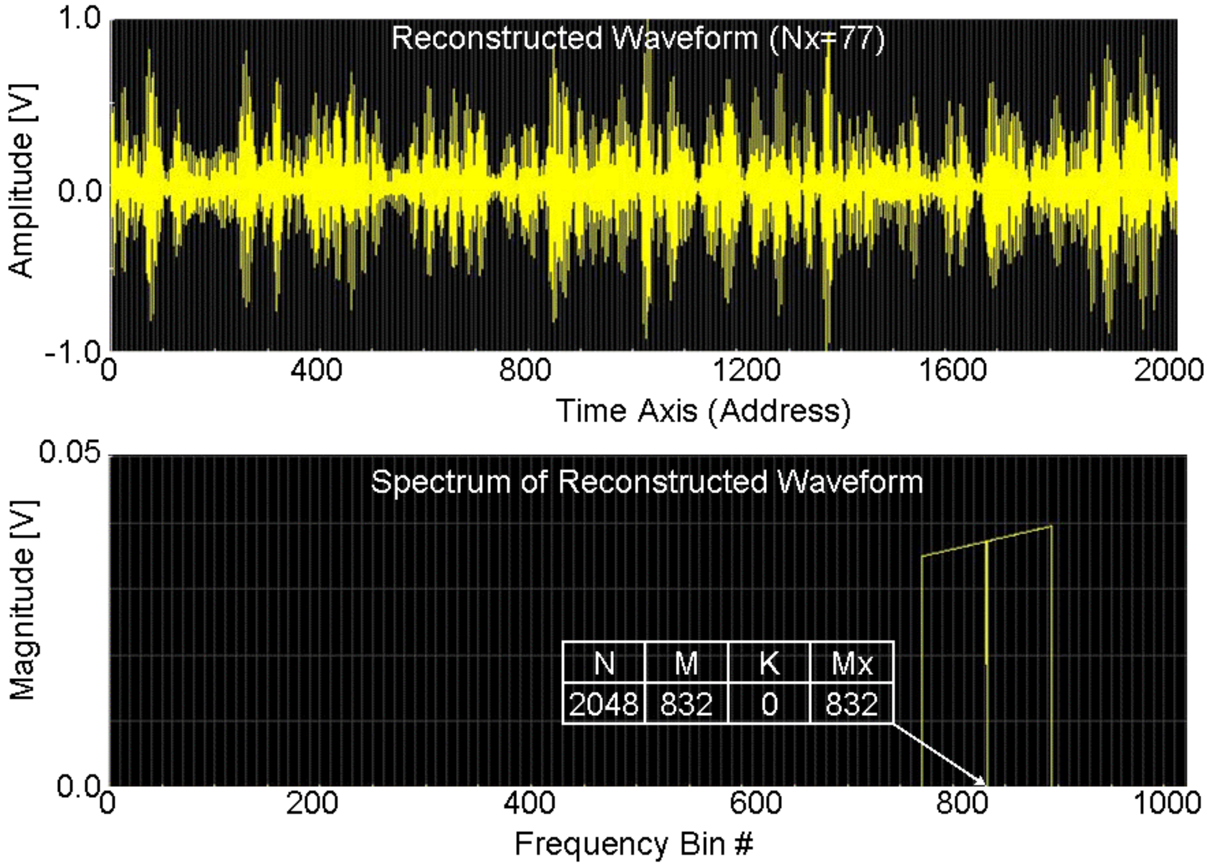

Figure 15: Reconstructed Waveform and Its FFT Spectrum

The informative portion of the signal is 123 lines out of 1024 bin locations in Figure 15. So you may want to reduce the number of sampling points to cover the area marked (b) in Figure 13 only. L=123 so that N could be reduced to 1024, 512 and 256 at the end. Figure 16 (a) shows the situation of N=2048 which is the case of Figure 15, and (b)(c)(d) show the cases of reduced-N.

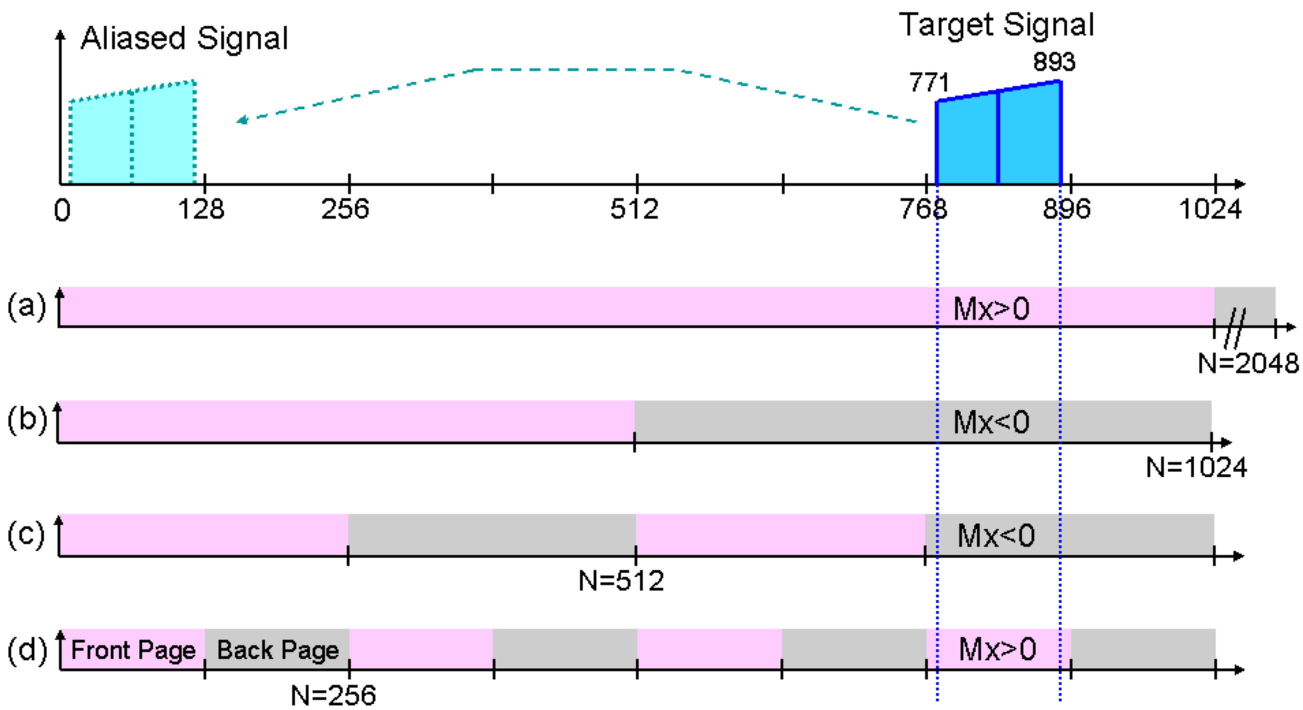

Figure 16: Reducing N to 1024, 512, 256

However, examining Figure 16 carefully, the target signal are located in the back page colored gray when N=1024 and 512 so that the spectrum would be expected to appear upside down.

The simulation results are shown in Figure 17, 18 and 20. You can find the slant level direction is opposite in Figure 17 and 18 so that they are reconstructed upside down as expected. The Mx is negative in these cases.

If you would like to turn over the reversed spectrum and correct it to straight, you can do it with the code example in List 1, which performs reversing the order of the spectrum allocation and playing complex conjugate operation to each one of the spectrum components.

Performing the spectrum order reversing, the spectra in Figure 17 and 18 are modified as Figure 19. The slant level response is corrected.

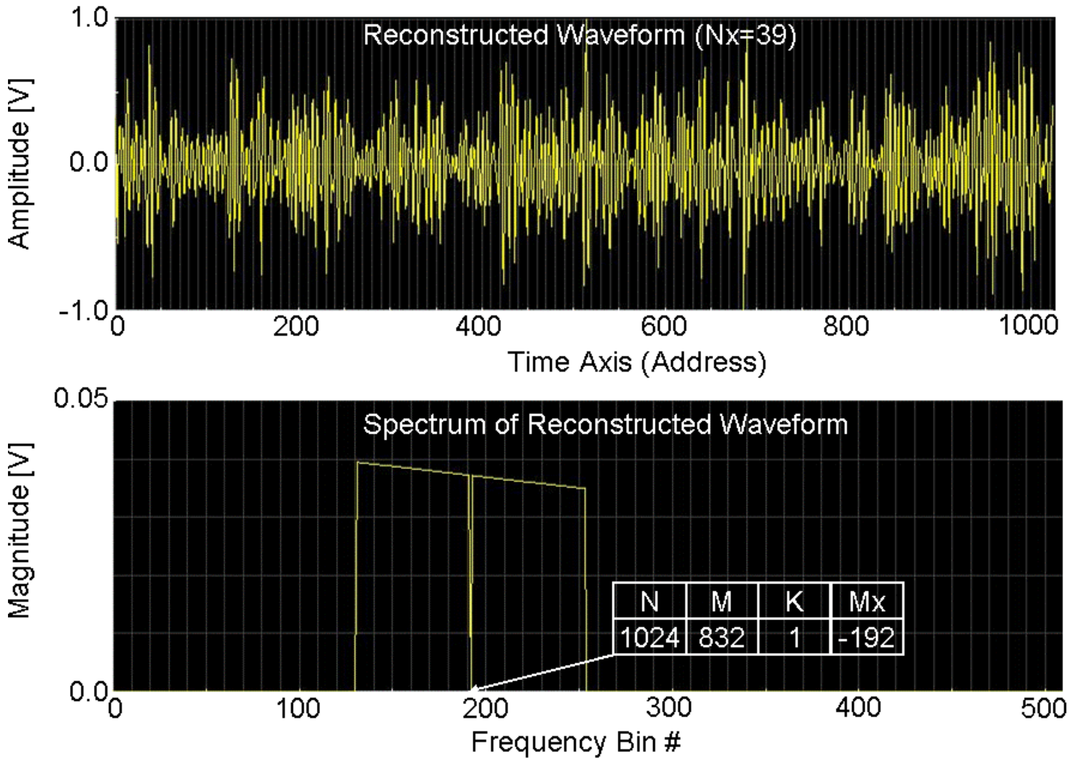

Figure 17: N=1024 Reconstructed Waveform and Spectrum

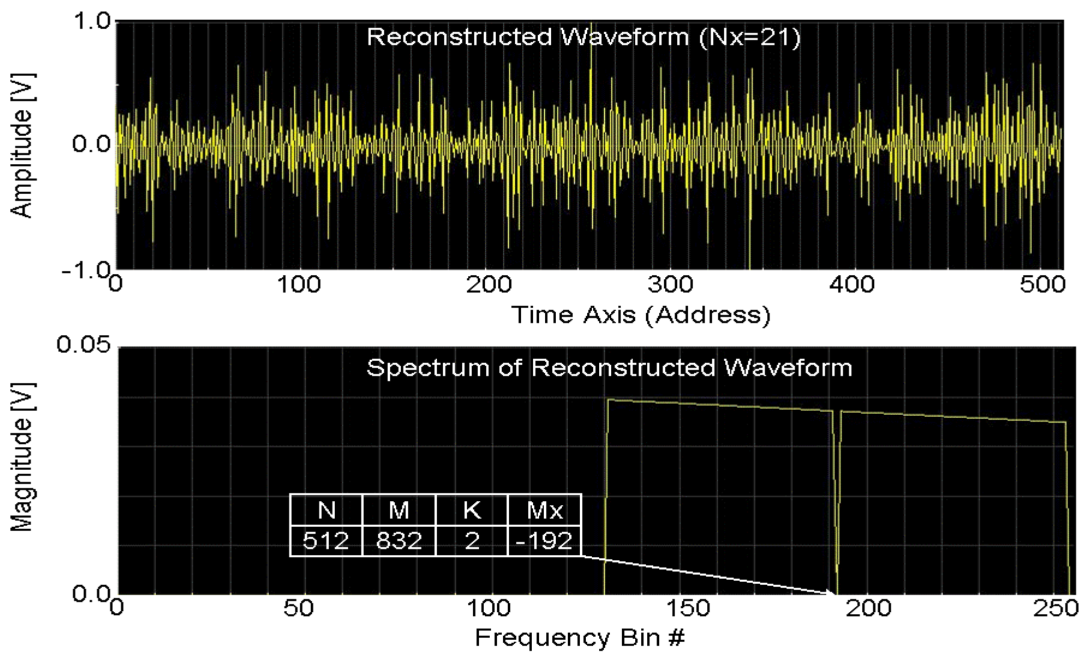

Figure 18: N=512 Reconstructed Waveform and Spectrum

```
INT                N,Nsp,Nx;           // N= # of Sampling Points
                                       // Nsp=# of Spectral Lines
COMPLEX            CTemp;              // Nx=Resolution Parameter
ARRAY_D            dVsmp,dWave;        // dWave: Reconst.Waveform
ARRAY_COMPLEX      CSp,CRev;           // CRev: Reversed Spectrum


dVsmp=DGT(“Tx”).getWaveform();         // Retrieve Sampler Data
N=dVsmp.size();                        // # of Sampling Points
Nsp=N/2;                               // # of Spectral Lines
Nx=39;                                 // 39@N=1024  21@N=512
DSP_SHUFFLE(dVsmp,dWave,Nx);           // Waveform Reconstruction
DSP_FFT(dWave,CSp,RECT);               // FFT Spectrum
CRev.resize(Nsp);                      // Reversed Spect Container
CRev[0]=CSp[0];                        // Save DC Bin
for (i=1;i<Nsp;i++) {                  // Bin#1 through (Nsp-1)
   CTemp=CSp[Nsp-i];                   // Reverse Order
   CRev[i].real()= CTemp.real();       // Complex Conjugate
   CRev[i].imag()=-CTemp.imag();       // For Phase Correction
}

```
List 1: Program Code to Turn Over Reversed Spectrum

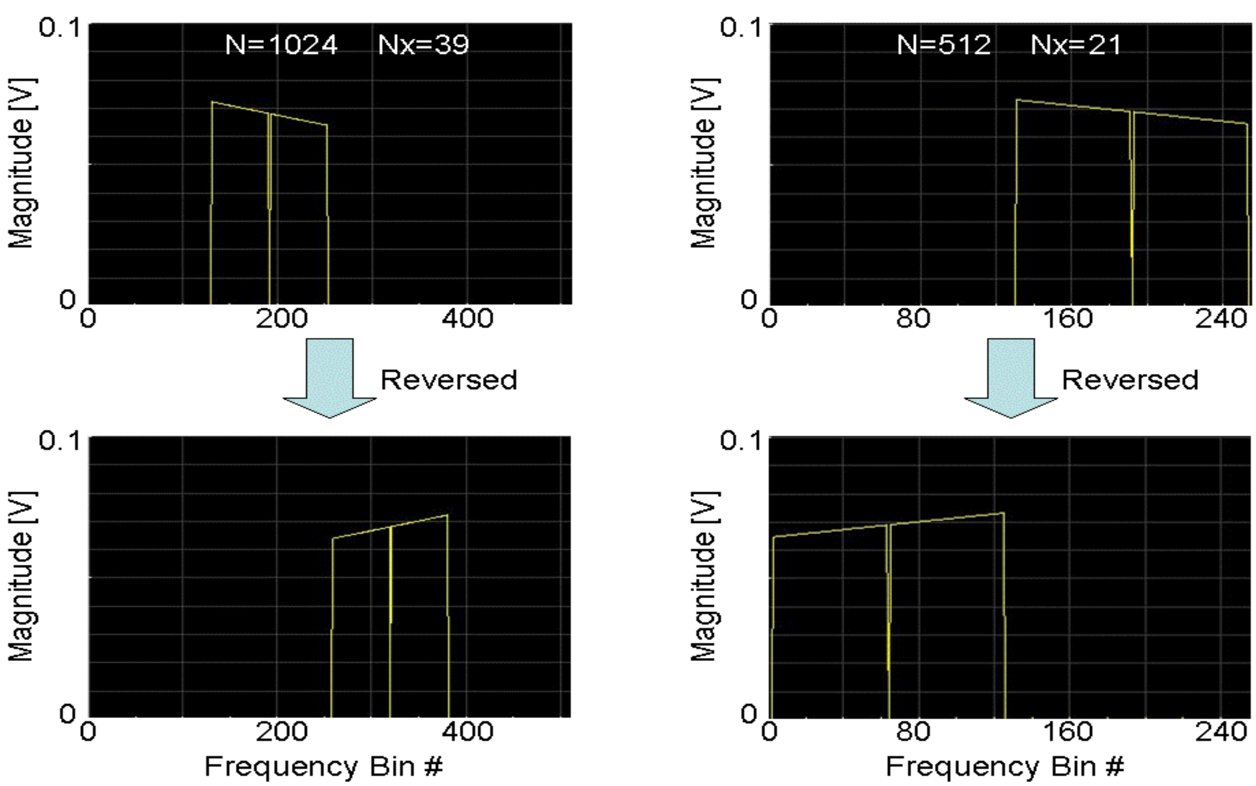

Figure 19: Spectra Turned-Over

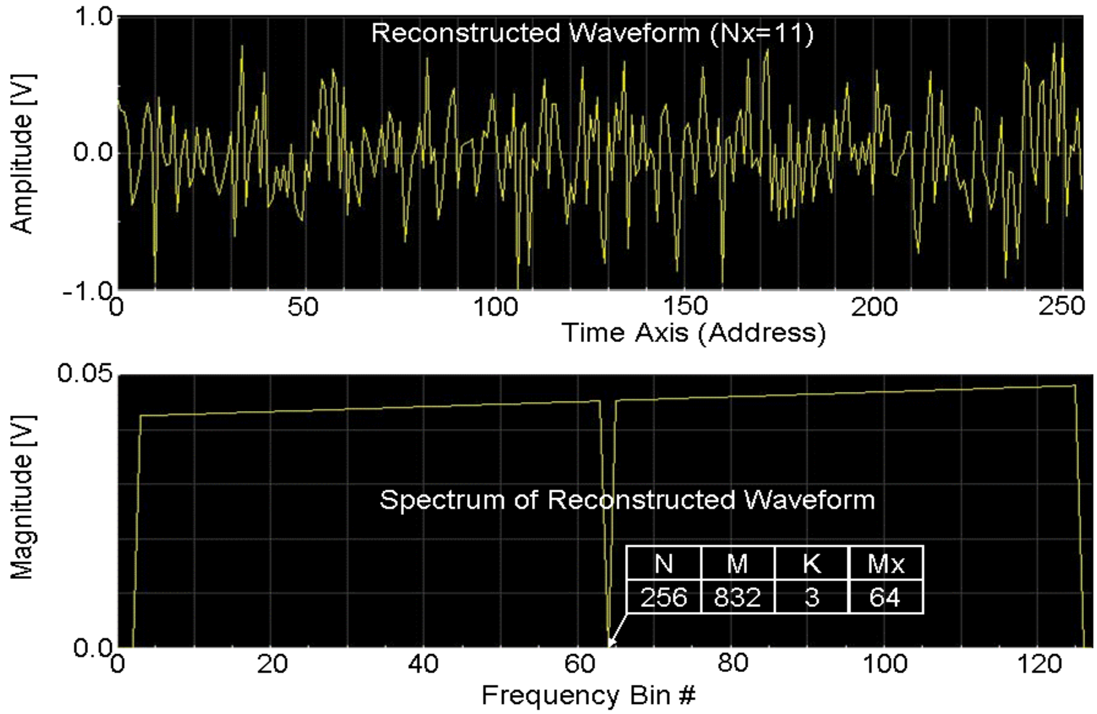

Figure 20: N=254 Reconstructed Waveform and Spectrum

Conclusion

Waveform samplers can test high frequency very wideband signals. The point is to set up measurement condition carefully not to conflict the aliased or folded spectrum components with each other in the baseband. In order to make a successful under-sampling for wideband multi-tone signals, the basic conditions are as follows.

1. $N > 2L$
2. $\triangle F = Nx * Fs / N$
3. Nx = Odd Integer
4. DSP_SHUFFLE(*Nx*) Reconstructs Informative Waveform and Spectrum
5. Localized Wideband Multi-tone: Full-band Capture or Just *L* Capture (Figure 13 (a)(b))
6. Reversed Spectrum in Just *L* Capture can be Corrected. (List 1)