



**Hideo Okawara's
Mixed Signal Lecture Series**

DSP-Based Testing – Fundamentals

Verigy Japan

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Preface

ADC and DAC are the most typical mixed signal devices. In mixed signal testing, analog stimulus signal is generated by an arbitrary waveform generator (AWG) which employs a D/A converter inside, and analog signal is measured by a digitizer or a sampler which employs an A/D converter inside. The stimulus signal is created with mathematical method, and the measured signal is processed with mathematical method, extracting various parameters. It is based on digital signal processing (DSP) so that our test methodologies are often called DSP-based testing. Test/application engineers in the mixed signal field should have thorough knowledge about DSP-based testing. FFT (Fast Fourier Transform) is the most powerful tool here. This corner will deliver a series of fundamental knowledge of DSP-based testing, especially FFT and its related topics. It will help test/application engineers comprehend what the DSP-based testing is and assorted techniques.

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Discrete Signal

In DSP-based ATE, analog signals are generated by arbitrary waveform generators (AWG) which contain D/A converters (DAC) inside, and analog signals are analyzed by digitizers or samplers which contain A/D converters (ADC) inside. DAC and ADC are also typical devices under test for mixed signal ATE.

DSP stands for digital signal processing. In other words it is discrete signal processing. When it comes to discrete signals, a signal consists of two dimensions (Figure 1). The x-axis corresponds to time, and the y-axis corresponds to magnitude which is usually voltage. When it comes to discrete time, the sampling theorem is important to know; while in terms of discrete voltage, quantization theorem is important to know.

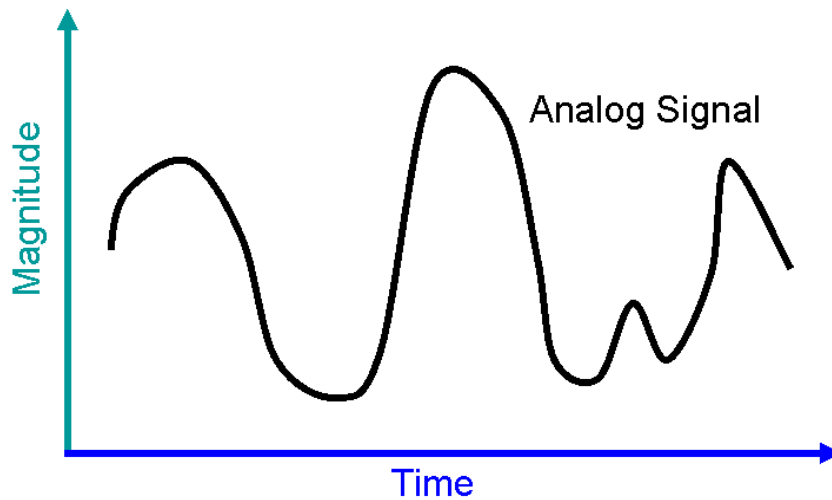


Figure 1. Analog Signal – Two Dimensions.

As a starter of this mixed signal test tutorial, let's review the sampling theorem and the quantization theorem, which every test/application engineer should comprehend very well.

Sampling Theorem

The sampling theorem states that if a signal is band-limited and the sampling frequency is greater than twice the signal bandwidth, it is possible to reconstruct the signal exactly (Figure 2). No information is lost. This rule is absolutely important for us to test an ADC and to use a digitizer for measurement. When the sampling rate is described as F_s , $F_s/2$ is called as Nyquist frequency and the frequency band up to $F_s/2$ is called the Nyquist band.

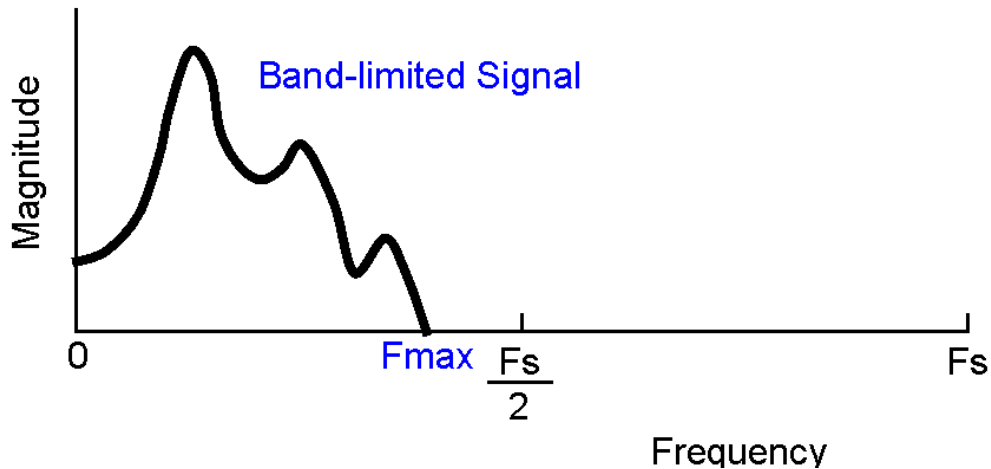


Figure 2. Sampling Theorem.

When we use a digitizer to measure analog signal, the input signal must be limited within the Nyquist band. The F_s must be selected as more than $2 \cdot F_{max}$, where F_{max} is the maximum frequency of the input signal. If you violate this rule, a signal greater than $F_s/2$ falls into the baseband (Nyquist band). This phenomenon is called "aliasing." You cannot separate the aliased signal from the original baseband signal. Digitizers provide several low-pass filters to eliminate high frequency noise and undesired signals. This filter is called an "anti-aliasing filter."

The higher the test signal frequency, the higher the digitizing frequency is required in digitizers for successful measurement. Then test signal frequencies easily exceed a digitizer's capability. Therefore another measurement resource is available, namely a waveform sampler. Aliasing is an undesirable phenomenon in regular digitizer applications, but a sampler utilizes it on purpose. Samplers have extremely wide input signal bandwidth, and usually no filters are integrated because samplers capture aliasing signals. In the discussion of samplers, the term "under-sampling" is often used to explicitly describe violation of the sampling theorem. "Under" seems to mean the sampling (digitizing) frequency is less than the target test signal frequencies. In terms of samplers, further discussion will be provided in a future issue of this go/semi newsletter.

Quantization Theorem

Let's move on to another axis. Approximating analog voltage into appropriate discrete levels is called quantization (Figure 3).

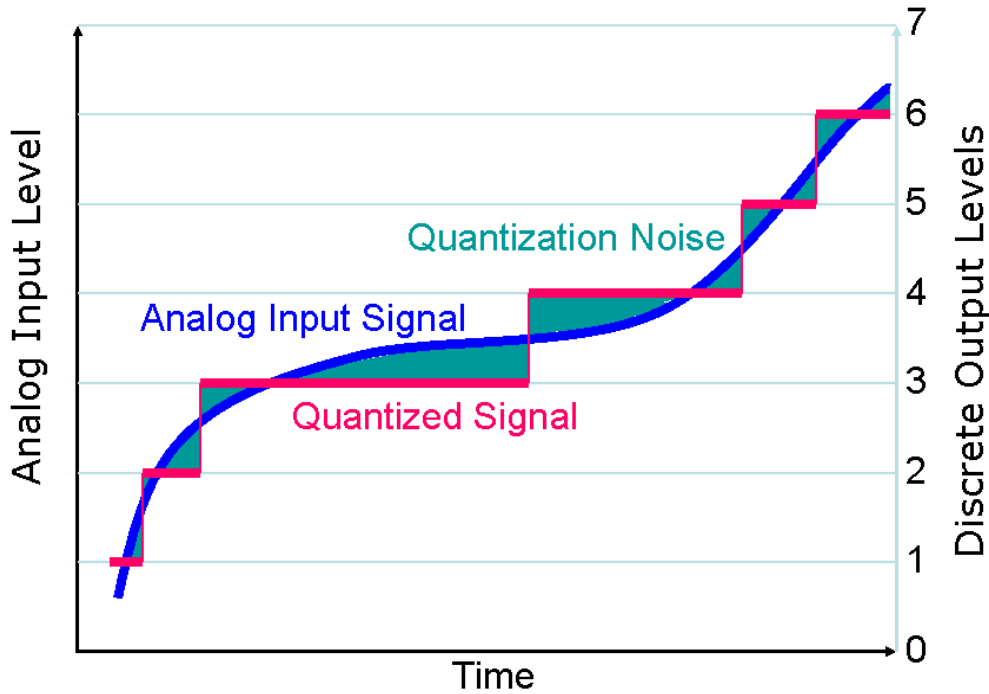


Figure 3. Quantization.

The most typical device for quantization is ADC. There is always quantization noise or errors existing in the approximating process. There is a very important equation in terms of quantization noise. In this section, let's derive the equation.

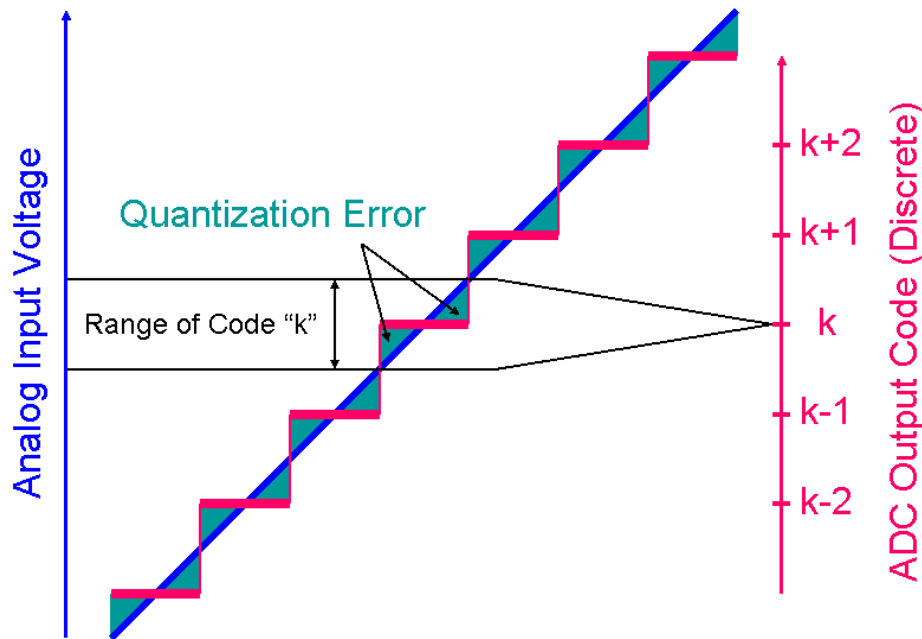


Figure 4. ADC Quantization.

Referencing Figure 4, an n -bit linear ADC contains 2^n codes from code 0 to $2^n - 1$. When a single code size is expressed as Q , an analog input voltage from $V_k - Q/2$ to $V_k + Q/2$ is represented as a code k , where V_k is the quantization level of the code k , and the center of the codes $(k-1)$ to k and k to $(k+1)$ transition thresholds. Figure 5 shows quantization error at a single code of ADC.

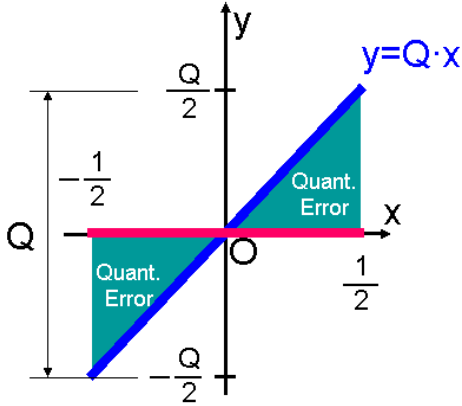


Figure 5: Quantization Error in a Code of ADC

The colored triangles represent quantization error so that the RMS noise is calculated as follows.

$$\begin{aligned}
 \text{Quant.Noise(RMS)} &= \sqrt{\int_{-\frac{1}{2}}^{\frac{1}{2}} y^2 dx} = \sqrt{\int_{-\frac{1}{2}}^{\frac{1}{2}} (Q \cdot x)^2 dx} = \sqrt{Q^2 \int_{-\frac{1}{2}}^{\frac{1}{2}} x^2 dx} \\
 &= \sqrt{Q^2 \cdot \left[\frac{x^3}{3} \right]_{-\frac{1}{2}}^{\frac{1}{2}}} = \sqrt{\frac{Q^2}{12}} = \frac{Q}{2\sqrt{3}}
 \end{aligned} \tag{1}$$

The full scale of the ADC input range is $2^n Q$. When the full scale of a sinusoidal waveform is applied to the ADC, the RMS signal amplitude can be described as follows:

$$\text{Signal_Amplitude(RMS)} = \frac{\text{Full_Scale}}{\sqrt{2}} = \frac{2^n \cdot Q}{\sqrt{2}} = \frac{2^n \cdot Q}{2\sqrt{2}} \tag{2}$$

SNR is the ratio of the RMS signal to RMS noise. Therefore the SNR base on the quantization is calculated as follows:

$$\begin{aligned}
 \text{SNR[dB]} &= 20 \log \frac{(\text{Signal_Amplitude(RMS)})}{(\text{Quant.Noise(RMS)})} = 20 \log \left(\frac{\frac{2^n \cdot Q}{2\sqrt{2}}}{\frac{Q}{2\sqrt{3}}} \right) = 20 \log \left(2^n \sqrt{\frac{3}{2}} \right) \\
 &\approx 6.02n + 1.76
 \end{aligned} \tag{3}$$

This is the most important equation in the ADC/DAC field – test/application engineers should learn this equation by heart! It defines the ultimate SNR value of an n-bit converter. For instance, when you have an excellent 8-bit ADC test, probably you would have approximately 48dB of SNR. However, if you get more than 50dB of SNR for the 8-bit device, there must be some mistake in the calculation process, or you may limit the measurement bandwidth by hardware or software. The noise power is proportional to the bandwidth.

When you measure the SNR of a device, you can calculate the effective number of bits (ENOB) of the device from the SNR value as follows.

$$ENOB[bits] = \frac{SNR[dB] - 1.76}{6.02} \quad (4)$$

This is a performance indicator of a converter device.