Question: Why is Random Jitter Always Modeled with a Gaussian Distribution?

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I always read that random jitter is Gaussian and unbounded. But why do we use a Gaussian distribution (and not another type)?

Answer:

The choice of the Gaussian distribution is not by chance or purely based on looking to the shape of a random jitter histogram and deciding it looked like a normal or Gaussian distribution. There is a mathematical principal that brings some insight into this choice; this principal is known as the Central Limit Theorem.

> Given N independent random variables X_i, we can add them in the following way:
>
> $$X=X_1+....+X_N$$
>
> This is a random variable with mean $\mu = \mu_1 + \mu_N$ and variance $\sigma^2 = \sigma^2_1 + ... \sigma^2_N$. The Central Limit Theorem states that under certain general conditions, the distribution F(x) for X approaches a Gaussian distribution with the same mean and variance.

The Central Limit Theorem tells us that if we have a system where the result of a given measurement is due to the contributions of a large number of independent events that are controlled by any kind of random distribution, then the distribution of the sum of these events will be Gaussian in nature.

As an example, take a crystal used for the reference clock of a system. The crystal is composed by several atoms in a crystalline structure where each atom will have the same response (with an unknown distribution) to external factors like temperature, etc. When looking at the measurement value for this crystal, we are interested in the jitter from the crystal over a certain period of time, this jitter value will be the result of the behavior of all the atoms in the crystal. We can then assume that the resulting measurement distribution will follow a Gaussian distribution because of the Central Limit Theorem.

Of course, the mathematics of a Gaussian distribution imply that the tails of the curve are infinite which we know is not true because it would not be possible in the real world. However, one needs to understand that the tails of the Gaussian distribution reduce to very low probability values very quickly which implies that the probabilities of events at the end of the tails will be computed in the "million of years" type of numbers. This is so far outside our area of interest that it makes little difference to using the Gaussian model on analyzing the worst case jitter scenario on a real data transmission system (e.g., how many failed bits in one day).

Another example on how to look to the central limit theorem and the infinite tails of a Gaussian distribution is the following:

> Take N dice, roll them and sum up the result. The distribution converges towards a Gaussian distribution, e.g., for N=10 it already looks pretty Gaussian. But notice that it is bounded because we have only 10 dice (the minimum is 10 the maximum is 60). To have an unbounded Gaussian distribution we would need an infinite number of die which is the requirement of the central limit theorem.

With random jitter we have a similar behavior: Jitter looks Gaussian but somehow it is still bounded, because the number of contributors is limited.

A good reference for more details on the Central Limit Theorem is: Athanasios, Papoulis and S. Unnikrishna Pillai “Probability, Random Variables and Stochastic Processes”.

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