Hideo Okawara's
Mixed Signal Lecture Series

DSP-Based Testing – Fundamentals 8
Under Sampling

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Preface to the Series

ADC and DAC are the most typical mixed signal devices. In mixed signal testing, the analog stimulus signal is generated by an arbitrary waveform generator (AWG) which employs a D/A converter inside, and the analog signal is measured by a digitizer or a sampler which employs an A/D converter inside. The stimulus signal is created with mathematical method, and the measured signal is processed with mathematical method, extracting various parameters. It is based on digital signal processing (DSP) so that our test methodologies are often called DSP-based testing.

Test/application engineers in the mixed signal field should have a thorough knowledge about DSP-based testing. FFT (Fast Fourier Transform) is the most powerful tool here. This corner will deliver a series of fundamental knowledge of DSP-based testing, especially FFT and its related topics. It will help test/application engineers comprehend what the DSP-based testing is and assorted techniques.

Editor's Note

For other articles in this series, please visit the Verigy web site at www.verigy.com/go/gosemi.

Under-sampling

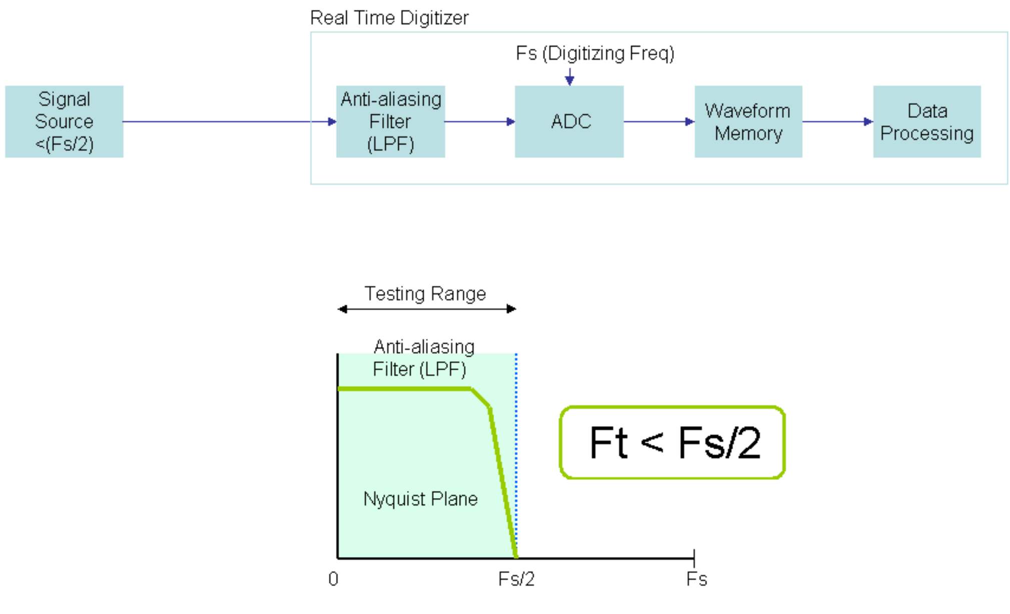
You may remember one of the fundamental theories discussed in the first article -- “Nyquist Theory.” If your signal is band-limited, when you would sample it with the frequency more than twice the maximum frequency of the band, all characteristic information of the signal is stored in the discrete time data stream. In other words, if the sampling frequency is lower than twice the bandwidth, something would be lost. This condition is called “under-sampling,” which is the theme of this article.

Digitizer and Sampler

In the V93000 SOC test system, there are two kinds of analog measurement instruments available. One is called a digitizer and the other is a sampler. Both of them employ A/D converters inside.

A digitizer takes care of signals that conform the Nyquist theory. A sampler measures signals that may exceed more than the half of the sampling frequency.

Figure 1 shows a typical block diagram of a digitizer. The input signal first goes through a LPF called “anti-aliasing filter”, and the band-limited signal is quantized or digitized by the A/D converter. The sampling frequency in the digitizer should be greater than twice the cut-off frequency of the anti-aliasing filter to conform to the Nyquist theory. The digitized data array is stored in the waveform memory. The data will be processed to derive required test parameters by various DSP operations in the tester controller. The input analog bandwidth is specified by the anti-aliasing filter and the real time sampling frequency of the ADC. In general today’s mixed signal testers employ the order of 100Msps 16-bit digitizer. Digitizing oscilloscopes employ the order of >1Gsps 8-bit digitizer.



Test signal should be localized in the Nyquist band.

Figure 1: Digitizer

Figure 2 shows a typical block diagram of a sampler. The difference with the digitizer is that there is no anti-aliasing filter integrated but the sampling head, or track-and-hold device, is employed at the front end. The input analog bandwidth is specified by the performance of the sampling head, which is usually several GHz. The input signal path should have adequate band-width.

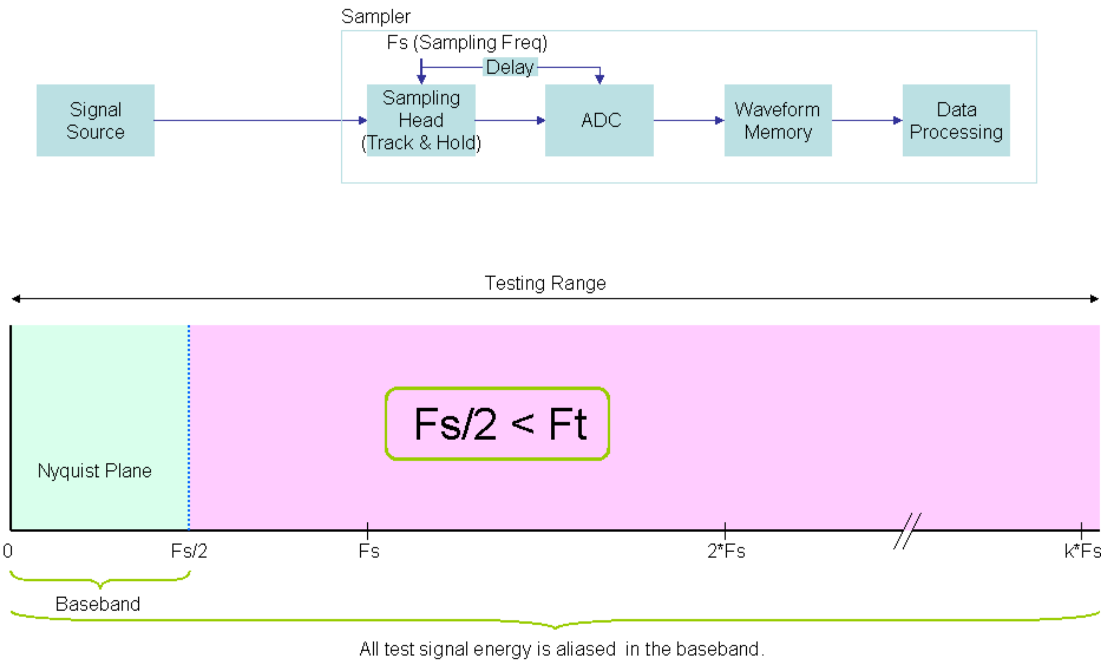


Figure 2: Sampler

Aliasing

"Aliasing" is often mentioned in the discussion about waveform digitizers and samplers. Aliasing is not a good thing in normal digitizing operations. It should be avoided for signal analyses. The Nyquist theory must be followed to the letter especially for signal to noise ratio (S/N). On the other hand, aliasing is the main player in the under-sampling condition. Figure 3 describes about the frequency domain of 8 points of sampled data (N=8). There are N/2=4 bin locations in the baseband area or the Nyqust plane. The bin locations more than N/2(=4) are folded and degenerated in the baseband page as depicted. This phenomenon is called "aliasing."

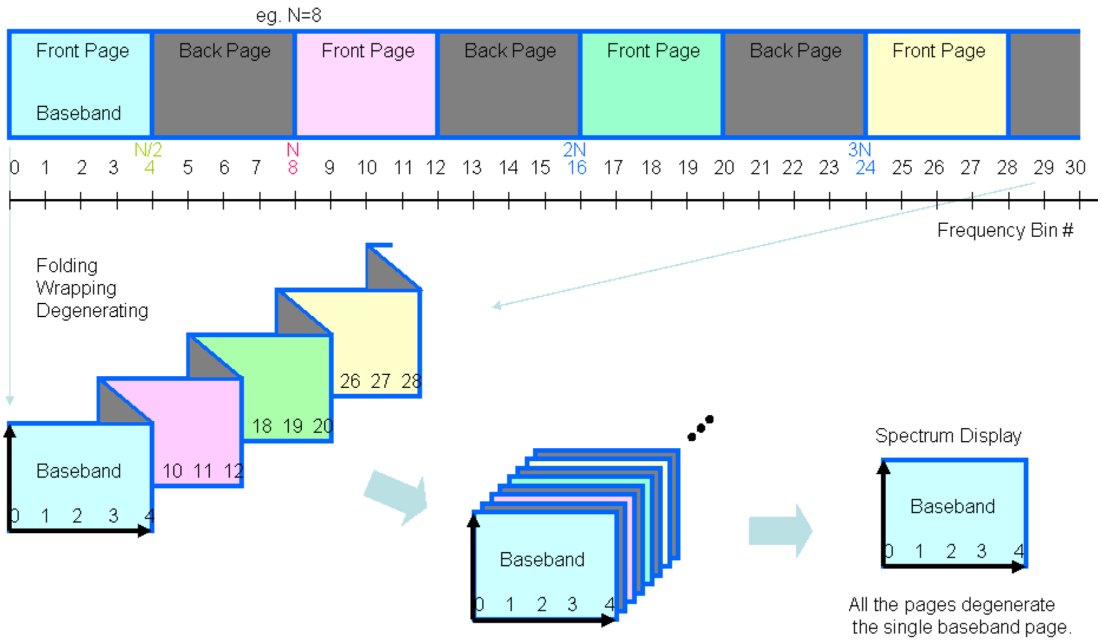


Figure 3: Aliasing

You can see the single baseband page as the frequency domain display, but this page actually contains entire information in higher pages and the display is the mixture of them. The planes labeled as "Front Page" in the picture are transparent on the baseband page in the normal direction, but the planes labeled as "Back Page" are transparent inside out. You can see the baseband page only, but you see mixture of all pages. Therefore once aliasing occurs, you cannot tell how the original spectrum looks without enough knowledge about the signal in advance.

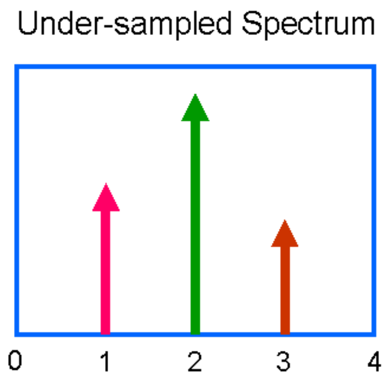


Figure 4: A Spectrum

When you see the spectrum as Figure 4 in under-sampling situation, you might think that the original spectral lines are located at the bins #25, 26, 27 as Figure 5.

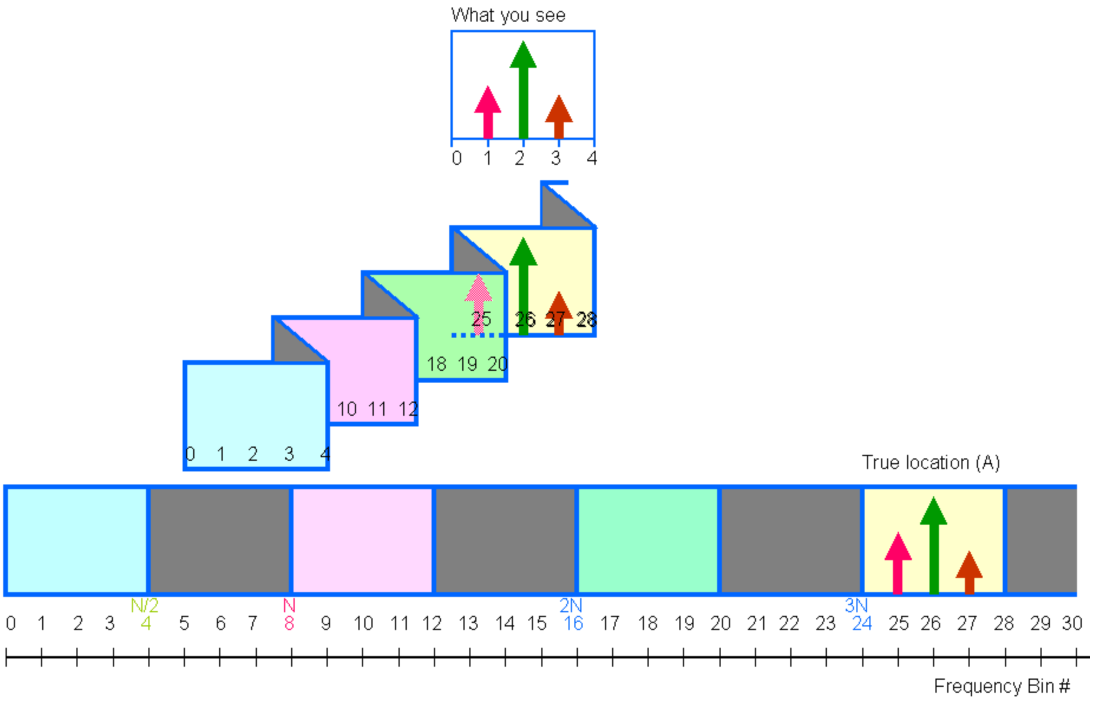


Figure 5: Original Signal Location (A)

However, the true signal location may be located at the bins #1, 19, 26 as Figure 6 shows. Both situations result in the same spectrum as Figure 4.

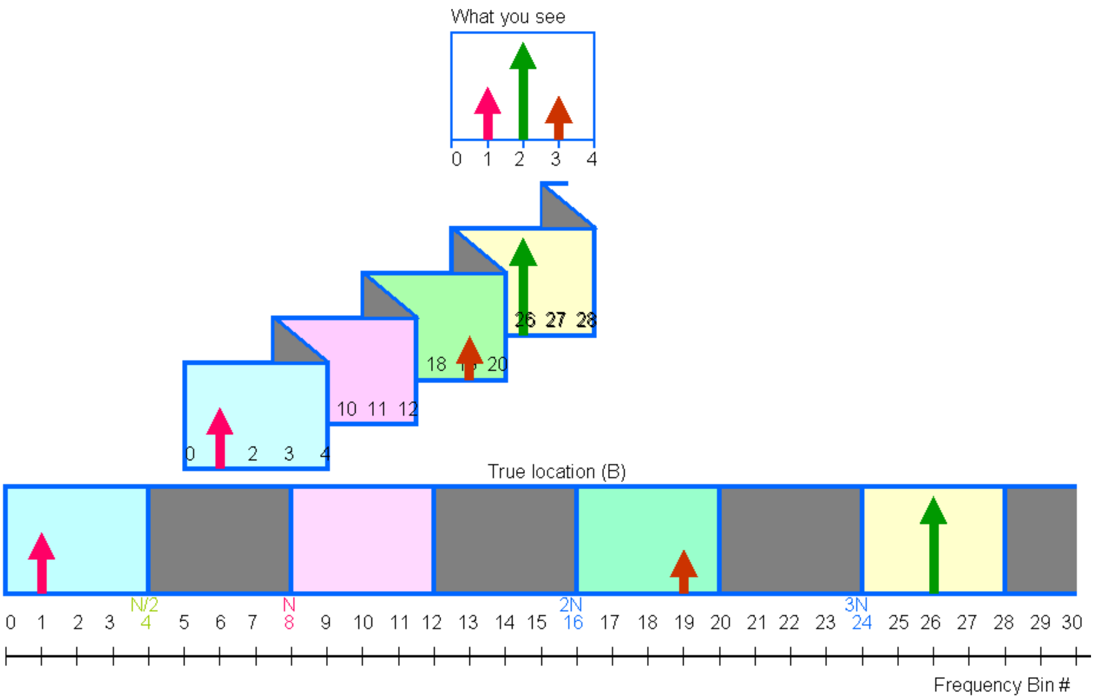


Figure 6: Original Signal Location (B)

Now that you understand what could happen when violating the Nyquist theory. When you do sampling that violates the Nyquist theory, it is called “under-sampling,” which consequently loses something. The frequency information is lost. However, in the mixed signal testing arena, the stimulus signal is created by you, and is supplied to the DUT. The signal is well known to you. Therefore you can locate which spectrum corresponds to the test signal you applied with a simple math system.

Coherent Condition

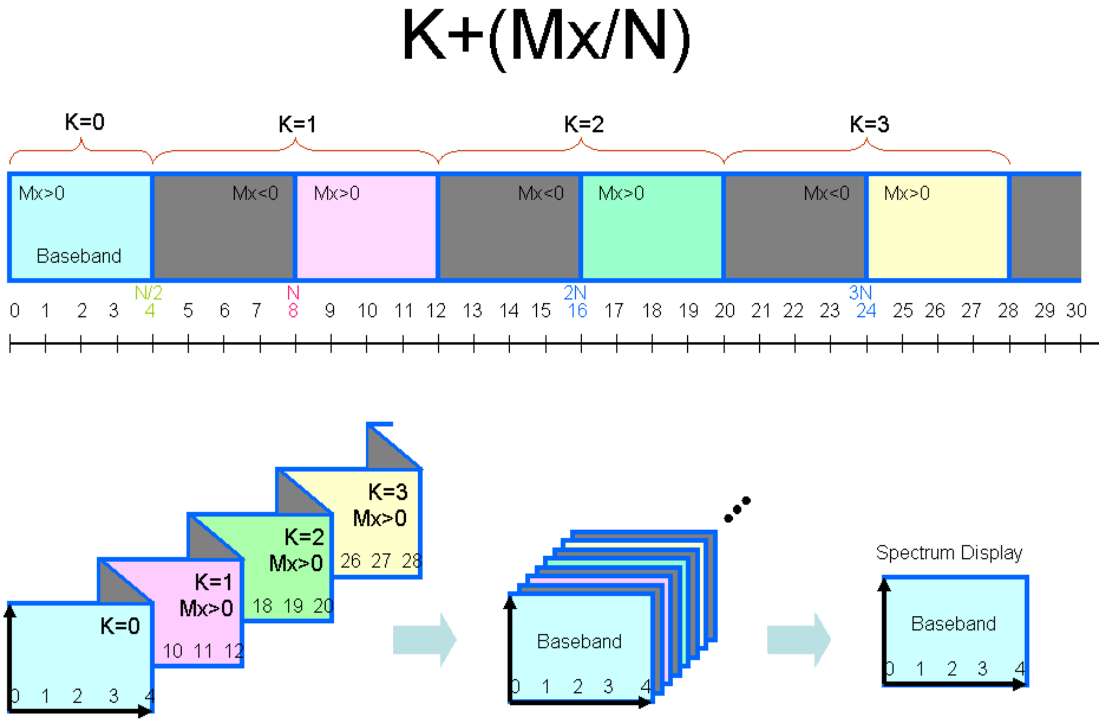
The coherent condition is the key to successful measurement in the DSP-based testing. It is described Ft/Fs=M/N, where Ft is the test signal frequency, Fs is the sampling frequency, M is the number of test signal cycles, and N is the number of data. When under-sampling, Ft>Fs/2 so that M>N/2. Consequently in the under-sampling situation, the coherent condition can be extended as follows.

$$\frac{Ft}{Fs} = \frac{M}{N} = K + \frac{Mx}{N} \quad (1)$$

Ft: Signal Frequency	M: Number of Cycles	K: Zone Number
Fs: Sampling Frequency	N: Number of Points	Mx: Aliased Bin Number

M and N must be integer numbers, and should be mutually prime. This rule is inherited to Mx and N combination. Mx should be between –N/2 and N/2. Usually N is 2^n for convenience of FFT so that Mx is an odd number.

The coherent equation (1) is related to the degenerated spectrum as Figure 7. The point is the aliasing bin number Mx, which is between –N/2 and N/2. If Mx>0, then the signal is located in the front page. If Mx<0, then the signal is located in the back page. Consequently |Mx| indicates where the aliased signal would fall in the baseband. When you make up your test condition with under-sampling, you may want to set up your spectrum appearance as you like. Usually Ft and N are settled at first so that you would have freedom to decide Fs and Mx. If you can control Fs, you can adjust Mx for your favorite location. You should keep in mind the relationship described in Figure 7. Generally speaking, a mixed signal tester must have at least two master clock domains. Probably the test frequency Ft would be based on one master clock domain. Then the sampling frequency Fs requires taking the other master clock domain for being precisely set up. Otherwise you could not settle your test condition coherently.



$$\frac{Ft}{Fs} = \frac{M}{N} = K + \frac{Mx}{N}$$

Figure 7: Coherent Equation vs Degenerated Spectrum

Waveform Reconstruction

There is a clock waveform whose frequency is Ft. Let's see a normal sampling (digitizing) situation. The signal is sampled by the 16 times higher sampling frequency Fs. (Fs=16Ft) A single clock waveform is digitized with 16 points of data (N=16) as Figure 10. The captured data automatically replicates the original clock waveform. In this case, the coherent equation is Ft/Fs=1/16 so that M/N=1/16.

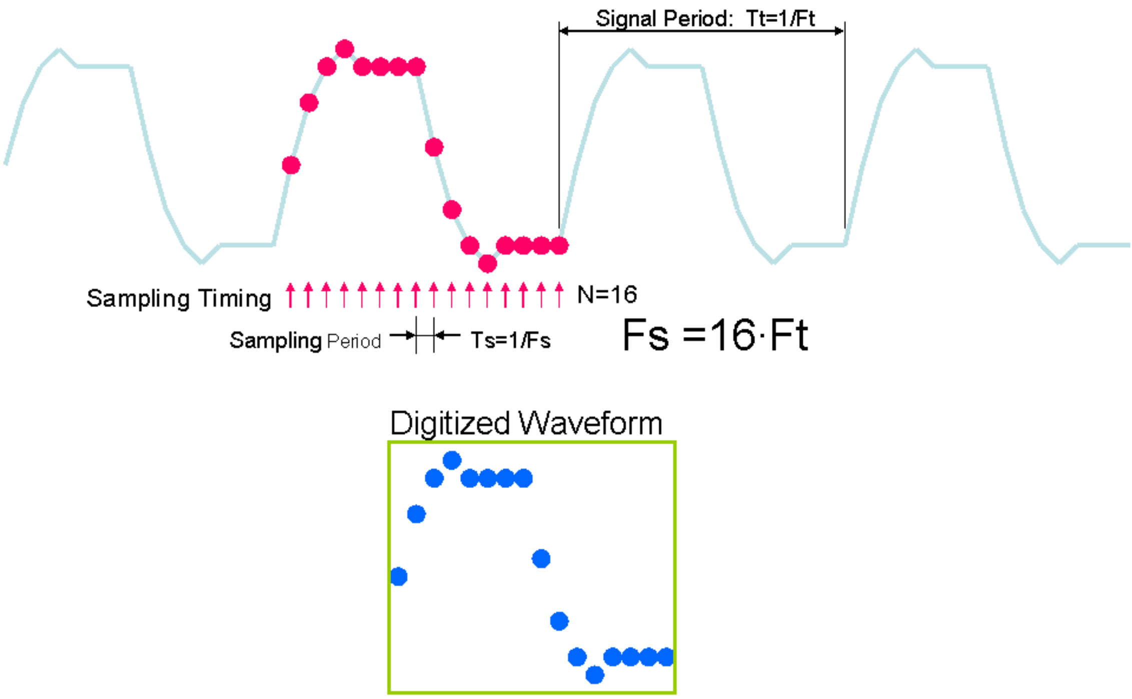


Figure 10: Normal Sampling / Digitizing (1)

The next digitizing situation is in Figure 11. The three cycles of the clock waveform is digitized with 16 points of data so that the coherent equation is Ft/Fs=3/16 and M/N=3/16. In this case the digitized data consists of 3 cycles of the primitive waveform. When you apply DSP_SHUFFLE(3 cycles), a single cycle of the original waveform can be replicated as Figure 11.

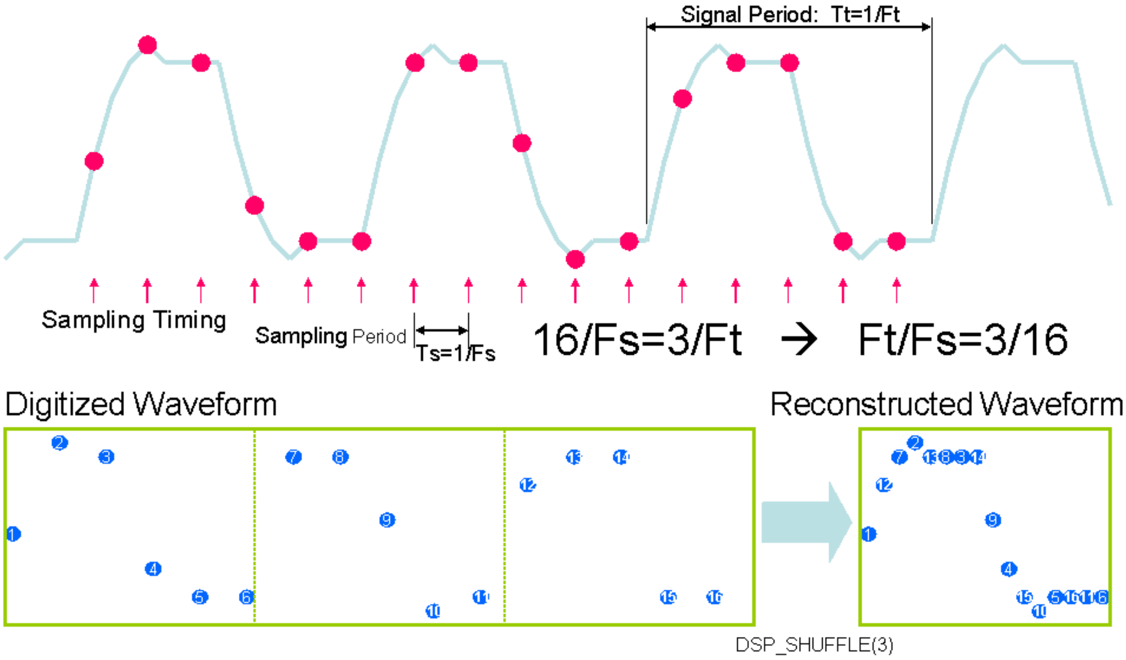


Figure 11: Normal Digitizing (2)

Let's move on to the under-sampling situation. (Figure 12) Each sampling point sweeps over the primitive clock waveform step by step. When 16-sampling completes, the sampled data directly replicates a single cycle of the original clock waveform. The coherent equation becomes Ft/Fs=2+1/16. Then M/N=K+Mx/N=2+1/16. Mx=1 here.

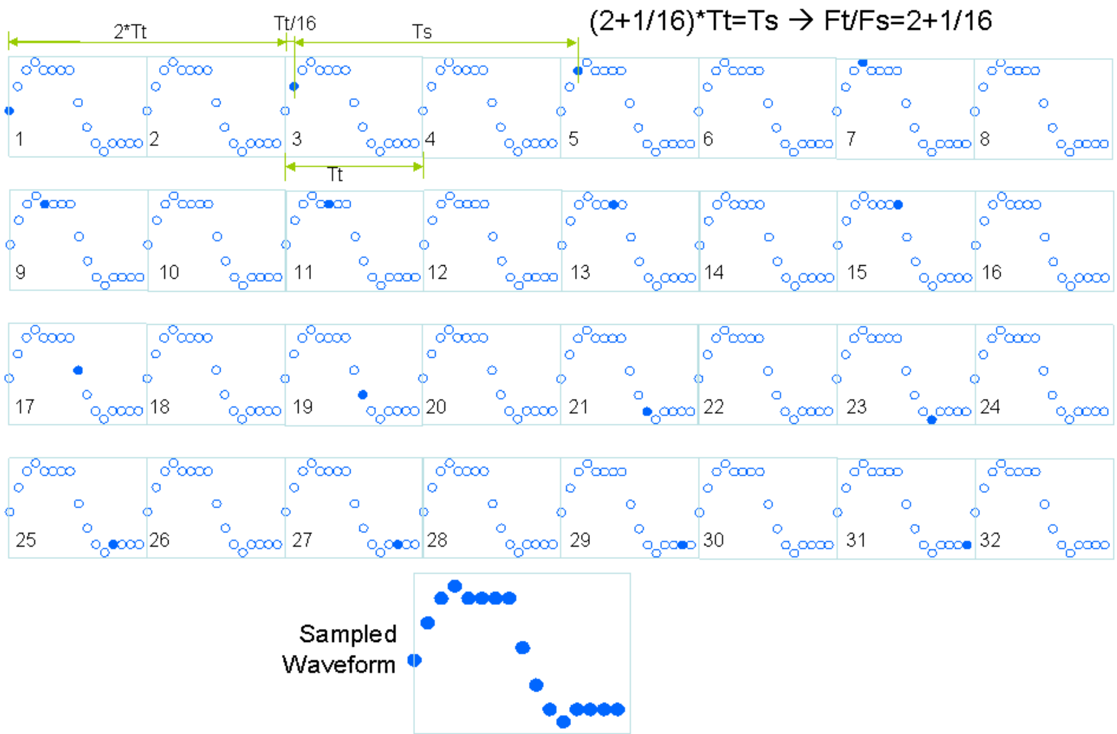


Figure 12: Under-sampling (1)

Next under-sampling in Figure 13 is Ft/Fs=2+3/16. So M/N=K+Mx/N=2+3/16. Then Mx=3. The three cycles of the primitive waveform is sampled so that DSP_SHUFFLE(3 cycles) can replicate a single cycle of the original waveform.

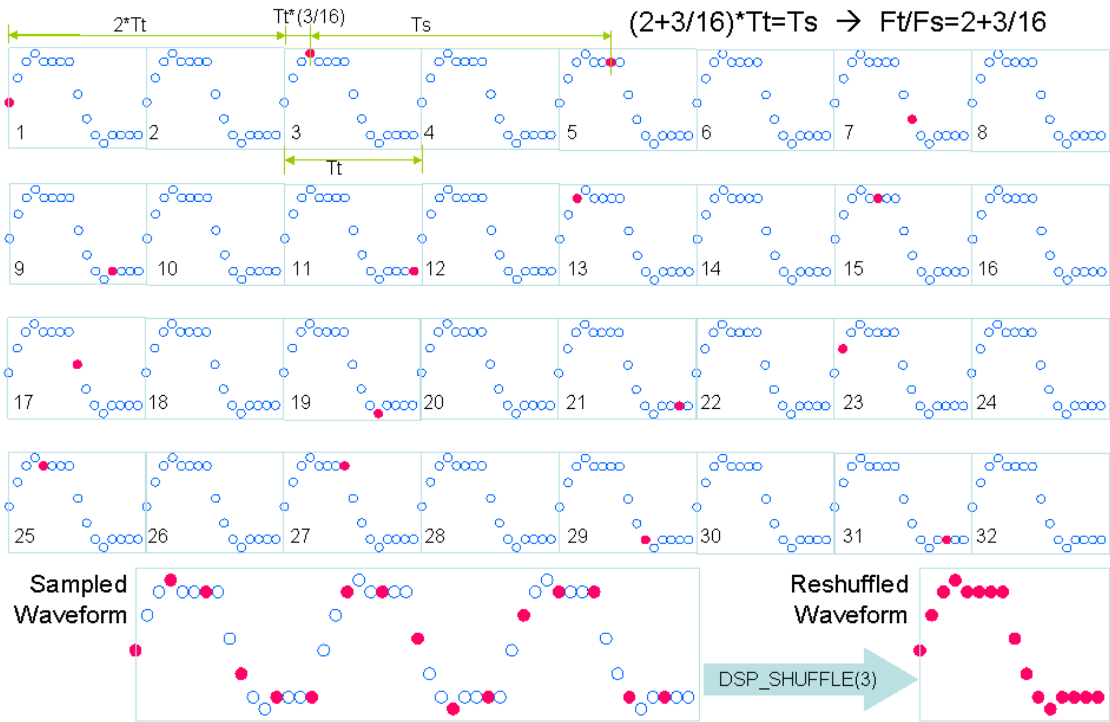


Figure 13: Under-sampling (2)

Figure 14 shows the case of M/N=K+Mx/N=2-1/16 so that Mx=-1. The sampled data directly replicates a single cycle of the original waveform, however it appears inside out.

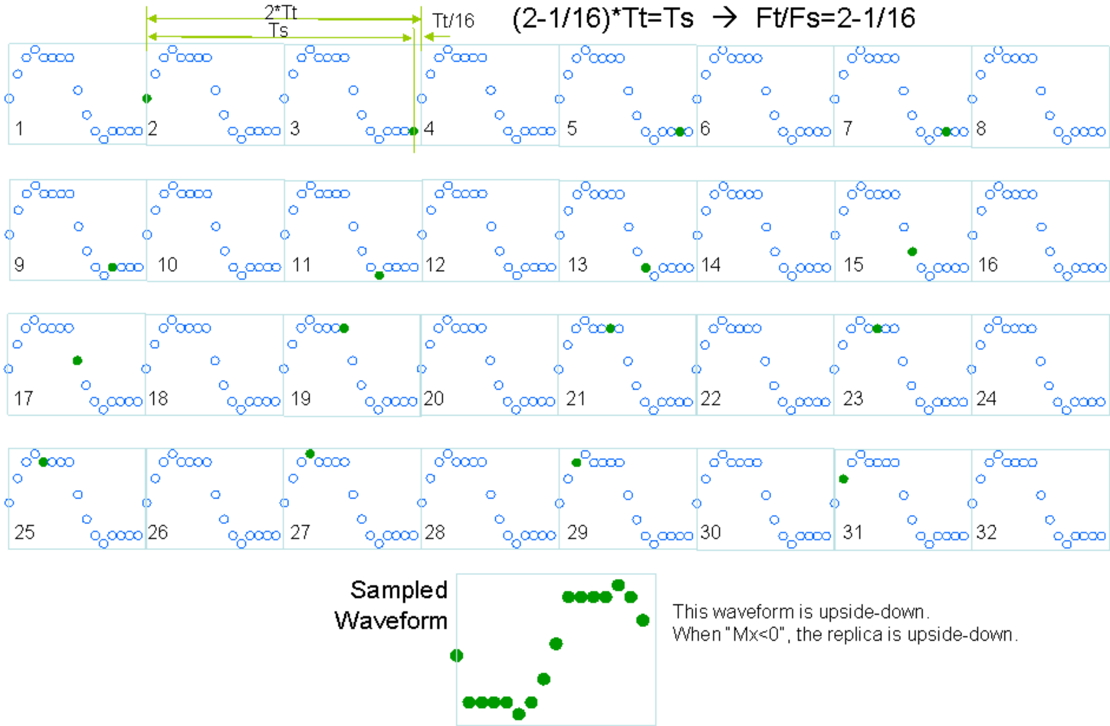


Figure 14: Under-sampling (3)

Figure 15 shows the case of M/N=K+Mx/N=2-3/16 so that Mx=-3. This is also the case of Mx<0. The sampled data captures 3 cycles of waveform so that DSP_SHUFFLE(3 cycles) replicates a single cycle of the original waveform. However, it appears inside out again.

As you already notice, when Mx is negative, the waveform is reconstructed inside out. It is no problem for the usual spectrum analysis. If you would feel uneasy with the waveform inside out, you may want to re-align the data array.

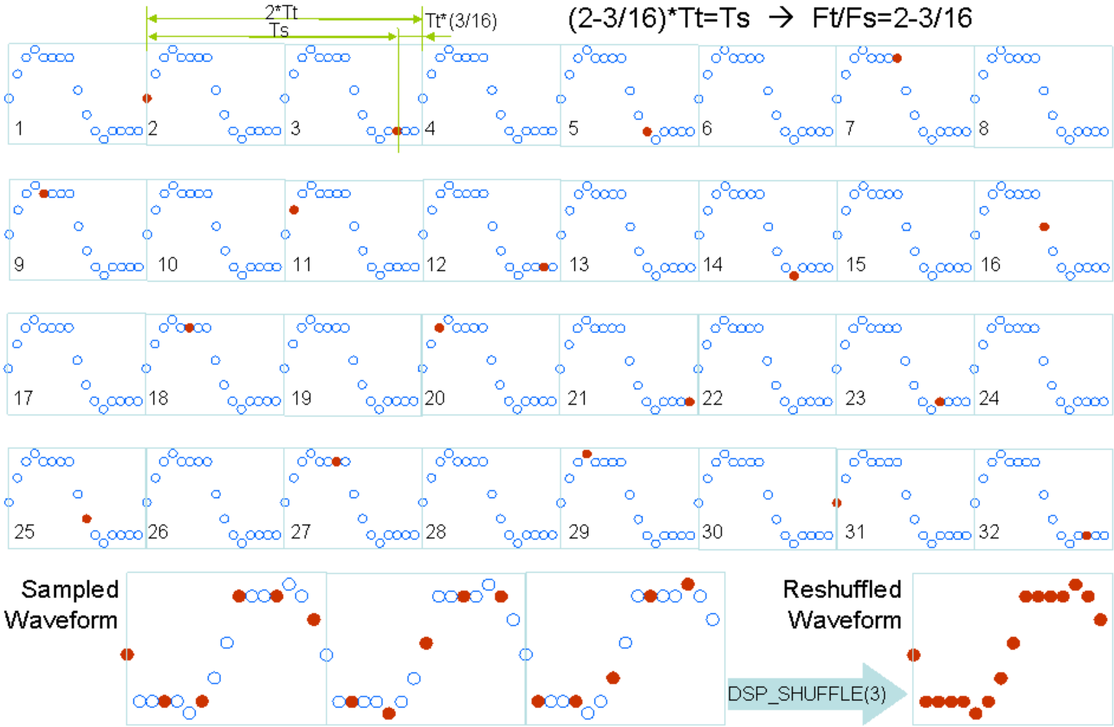


Figure 15: Under-sampling (4)