Smith Chart Tuning, Part I

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Abstract

Simple rules of Smith Chart tuning will be presented, followed by examples. The goal is for the reader to be able to identify correct tuning topologies *by inspection*.

1 Impedance

Impedance Z has real resistance R and imaginary reactance X :

$$Z = R + jX \tag{1}$$

In practice we will use both Impedance and Admittance extensively while tuning, though in the next few sections, we will only discuss Impedance to keep the flow of thought focused. Since there is a duality between Impedance and Admittance, the reader should be able to quickly grasp the analogous treatment of Admittance later in the article.

2 X, L, C

While tuning with the Smith Chart, reactance X is indicated on the chart, but the user usually works with inductance L and capacitance C. The relationship of X to L and C is:

$$X_L = \omega L \tag{2}$$

$$X_C = \frac{-1}{\omega C} \tag{3}$$

where

$$\omega = 2\pi f \tag{4}$$

and f is frequency. X is proportional to L, but X is *inversely* proportion to C. The reader should note this difference. Since the Smith Chart is plotted in X values, care must be taken when considering whether to use a smaller or larger capacitor due to this inverse relationship between X and C.

3 S-parameters

The Smith Chart was invented by Phillip Smith of Bell Labs in 1939 to deal with transmission lines. Today, it is also associated with S-parameters invented by K. Kurokawa [2] in 1965. Neither transmission line nor S-parameter theory is covered in depth in this article. A classic treatment of S-parameter theory is cited in the bibliography [1].

4 Reflection Coefficient

For the purpose of this article, it is enough to understand that S-parameter theory is based on the finite speed of light

$$c = \frac{1}{\sqrt{\epsilon_r}} \cdot 3 \cdot 10^{10} \text{ cm/sec} \tag{5}$$

where ϵ_r is relative dielectric constant of the medium. For FR4 material commonly used in PCB construction, $\epsilon_r = 4.5$, so the speed of light is approximately half of the speed in vacuum. A signal at $t_0 = 0$ which begins at the Network Analyzer, does not simultaneously appear at the load. The signal power actually flows in a power wave toward the load and reaches it at a delayed time $t_{Delay} = t_{Cable} + t_{Trace}$, and if the load impedance does not match the system impedance, some power is reflected back in a power wave traveling in the opposite direction while some of it is absorbed by the load. This reflected power is calculated by the equation for Reflection Coefficient:

$$\Gamma_{11} = \frac{Z_L - Z_0}{Z_L + Z_0} \tag{6}$$

where Z_L is the load impedance and Z_0 is the system reference impedance which is traditionally set to 50Ω, but can be 75Ω for TV applications, and 100Ω for differential systems. Since the value of system reference impedance Z_0 is not universal, we will henceforth use normalized impedance $Z = \frac{Z_L}{Z_0}$. The goal is to find a network which eliminates reflected power, i.e., $\Gamma_{11} \to 0$.

5 Smith Chart and Impedance

The normalized version of equation (6) is

$$\Gamma_{11} = \frac{1 - Z}{1 + Z} \tag{7}$$

Substituting equation (1) into equation (7) and using rules to manipulate complex expressions, we obtain

$$\Gamma_{11} = \Gamma_r + j\Gamma_i \tag{8}$$

$$\Gamma_r = \frac{R^2 + X^2 - 1}{(R+1)^2 + X^2} \tag{9}$$

$$\Gamma_i = \frac{2X}{(R+1)^2 + X^2} \tag{10}$$

The Smith Chart is generated by utilizing Equations (9) and (10) to plot impedance Z contours on the Γ_{11} plane. For passive impedance ($0 \leq R$) the $Z \rightarrow \Gamma_{11}$ mapping distorts the entire rectangular passive Z half plane into a compact circle in the Γ plane. $R = \infty$ now is at the right edge of the circle, and both $X = +\infty$ and $X = -\infty$ are bent over to meet the point $R = \infty$. While the fact that three infinities coming together at a single point can invoke deeply spiritual feelings, one important practical effect of this mapping is to conveniently convert an infinite half-plane into a compact finite area (figure 1).

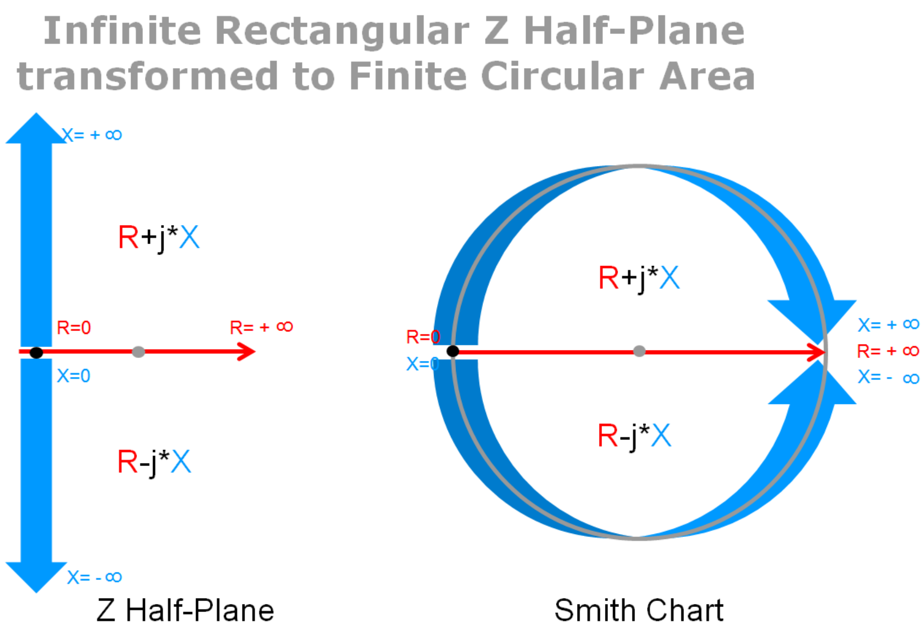

Figure 1: Z plane to Γ plane

Comparing the Z half plane to the Smith Chart circle, we see some interesting features. The horizontal red line segment which bisects the Smith Chart circle is pure resistance R. It is zero on the left endpoint and ∞ on the right endpoint, and the exact center is the system reference impedance Z_0 normalized to 1. The upper half circle has +X reactance and the lower half circle has -X reactance. From equations (2) and (3), this implies

> Insight Number 1: The upper half circle is inductive, while the lower half circle is capacitive.

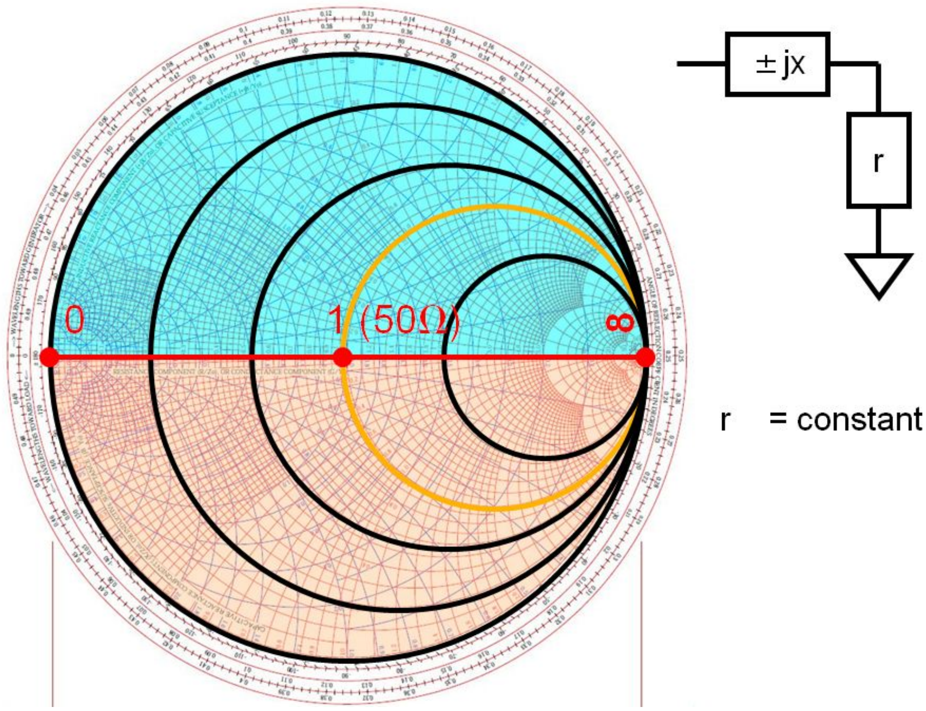

Figure 2: Series Tuning Circles

Now, let us choose a load impedance with some fixed R values while allowing X to vary $\pm\infty$. Not surprisingly, the result is a group of circles inside the circular Smith Chart. They all intersect at $R = \infty$ point on the right side of the Smith Chart. Impedance Z is a pure resistance R along the red horizontal line. Adding series +X will move Z into the upper half Smith Chart, while adding -X will move Z into the lower half Smith Chart. As $X \rightarrow \infty$ or $X \rightarrow -\infty$, Z bends toward the R $= \infty$ point on the right. However, among these circles, the circle drawn in yellow is very special. It corresponds to R=1 and goes through the center of the Smith Chart. Let us name this circle the "Golden Circle". Its impedance is $Z_{gold} = 1 \pm X$. Notice that adding a single pure reactance $X_{tune} = \mp X$ in series results in

$$Z_{gold} + X_{tune} = (1 \pm X) \mp X = 1 \tag{11}$$

But Z = 1 is the system reference impedance, so impedance match is achieved. This is a very important result. It means that regardless of where we begin our RF matching effort, we must eventually end on the Golden Circle, and the final tuning step moves along the Golden Circle toward match.

Insight Number 2: The last tuning step is on the Golden Circle

If by luck and coincidence, Z_{load} falls on the Golden Circle, then tuning for a match can be done with a single series element. Of course, engineers are usually very unlucky, so Z_{load} is rarely on the Golden Circle. Fortunately, there is another fascinating property of the Smith Chart which we will now discuss. We have studied the Smith Chart and impedance Z = R + jX, where R and X are in series. Now, we will study the Smith Chart where R and X are in parallel. In this case. it is more convenient to use the concept of Admittance. To understand why, let us quickly recall that for two series resistors, the total resistance is simply the sum $R_{tot} = R_1 + R_2$. But for parallel resistors, the total resistance is $\frac{1}{R_{tot}} = \frac{1}{R_1} + \frac{1}{R_2}$). If we use $\frac{1}{R}$ as a fundamental entity, then putting elements in parallel results in a simple sum. In an analogous way, since Admittance $Y = \frac{1}{Z}$ the mathematics of putting elements in parallel are simplified.

6 Admittance

$$Y = G + jB \tag{12}$$

where G is conductance and B is susceptance.
Impedance and Admittance are inversely related:

$$Y = \frac{1}{Z} \tag{13}$$

7 Y, G, B

When using admittance G and B, the relationships to L and C are:

$$B_L = \frac{-1}{\omega L} \tag{14}$$

$$B_C = \omega C \tag{15}$$

8 Smith Chart and Admittance

Combining Equations (13) and (6)

$$\Gamma_{11} = -\frac{1-Y}{1+Y} \tag{16}$$

Substituting equation (12) into equation(16) and using rules to manipulate complex expressions we obtain the Admittance equations for Smith Chart

$$\Gamma_r = \frac{-(G^2 + B^2 - 1)}{(G+1)^2 + B^2} \tag{17}$$

$$\Gamma_i = \frac{-2B}{(G+1)^2 + B^2} \tag{18}$$

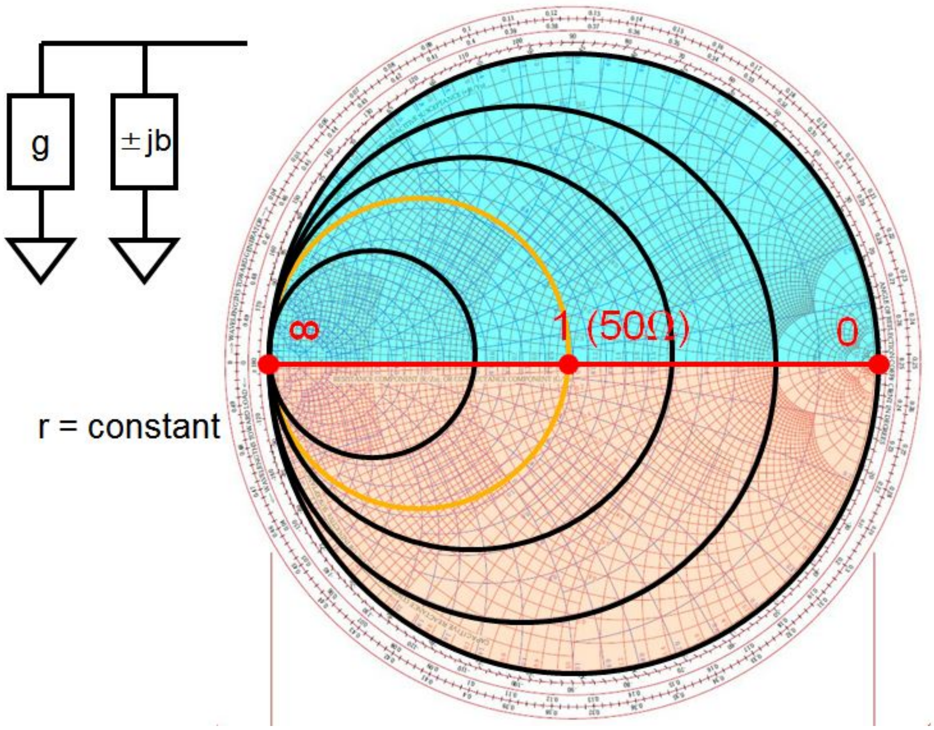

Figure 3: Shunt Tuning Circles

Now, let us choose a load admittance with some fixed G values while allowing B to vary ±∞. Not surprisingly, the result is a group of circles inside the circular Smith Chart. They all intersect at the $G = \infty$ point on the left side of the Smith Chart. Admittance Y is a pure conductance G along the red horizontal line segment. Adding shunt -B will move Y into the upper half Smith Chart, while adding shunt +B will move Y into the lower half Smith Chart. As $B \to \infty$ or $B \to -\infty$, Z bends toward the G = ∞ point on the left. However, among these circles, the circle drawn in yellow is very special. It corresponds to G=1 and goes through the center of the Smith Chart. Let us name this circle the "Golden Circle". Its impedance is $Y_{gold} = 1 \pm B$. Notice that adding a single pure susceptance $B_{tune} = \mp B$ in parallel results in

$$Y_{gold} + Y_{tune} = (1 \pm B) \mp B = 1 \tag{19}$$

But Y = 1 is the system reference admittance, so admittance match is achieved. This is a very important result. It means that regardless of where we began our RF matching effort, we must eventually end on the Golden Circle, and the final tuning step moves along the Golden Circle toward match.

After reading section 5 and section 8 did the reader receive the eerie feeling of what in the French language is called *Déjà vu* ? If so, this is another elegant aspect of the Smith Chart transformation, which is the complete duality between Impedance and Admittance. The mathematics of Impedance and Admittance in the Smith Chart possess symmetry, and the graphical behavior of one is a 180^o rotation of the other. In the next section, the reader will use this duality to perform matching.

9 Two Steps and Three Rules

Now we are ready to learn how to perform tuning with the Smith Chart. Let us consider an arbitrary passive load point Z_{Load} (red dot) on the Smith Chart. Two tuning circles pass through it, one Impedance, one Admittance. These are shown in black dotted lines below.

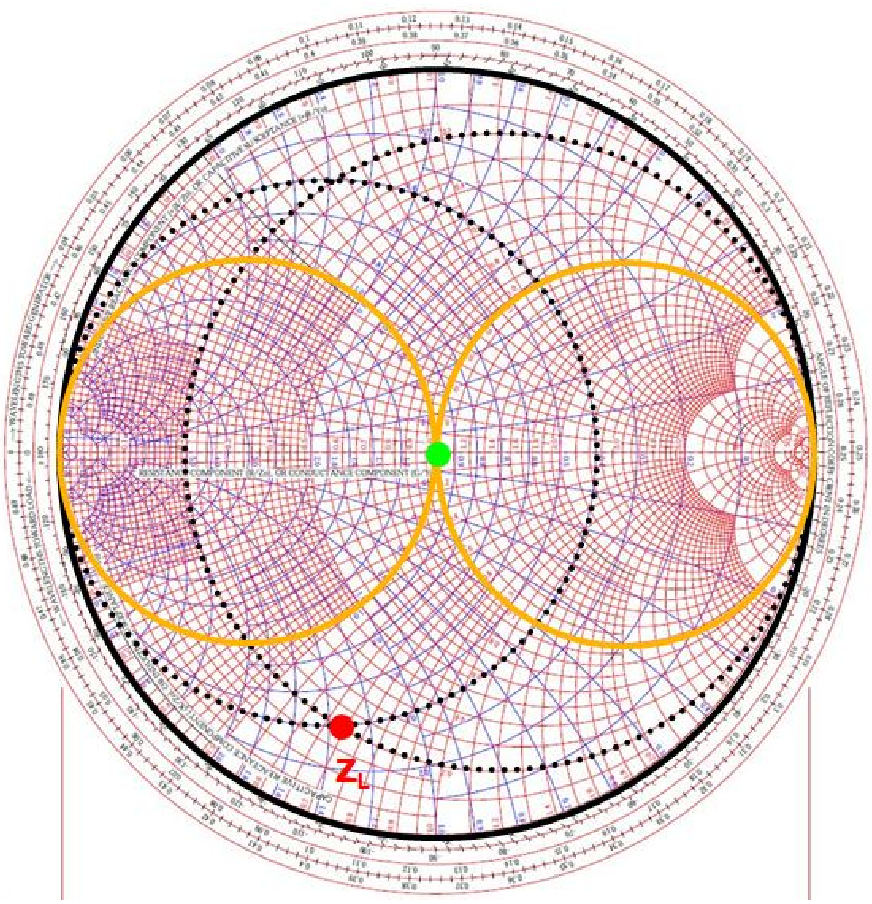

Figure 4: Find tuning circles and Golden Circles

Follow the dotted line paths from Z_{Load} (red dot) to Golden Circles. Then move along Golden Circles to the Smith Chart center (green dot).
By inspection, we discover four possible tuning paths to the center of the Smith Chart. Visualizing these paths is the first stage. For the second stage of obtaining more details of circuit topology (series/shunt) and type

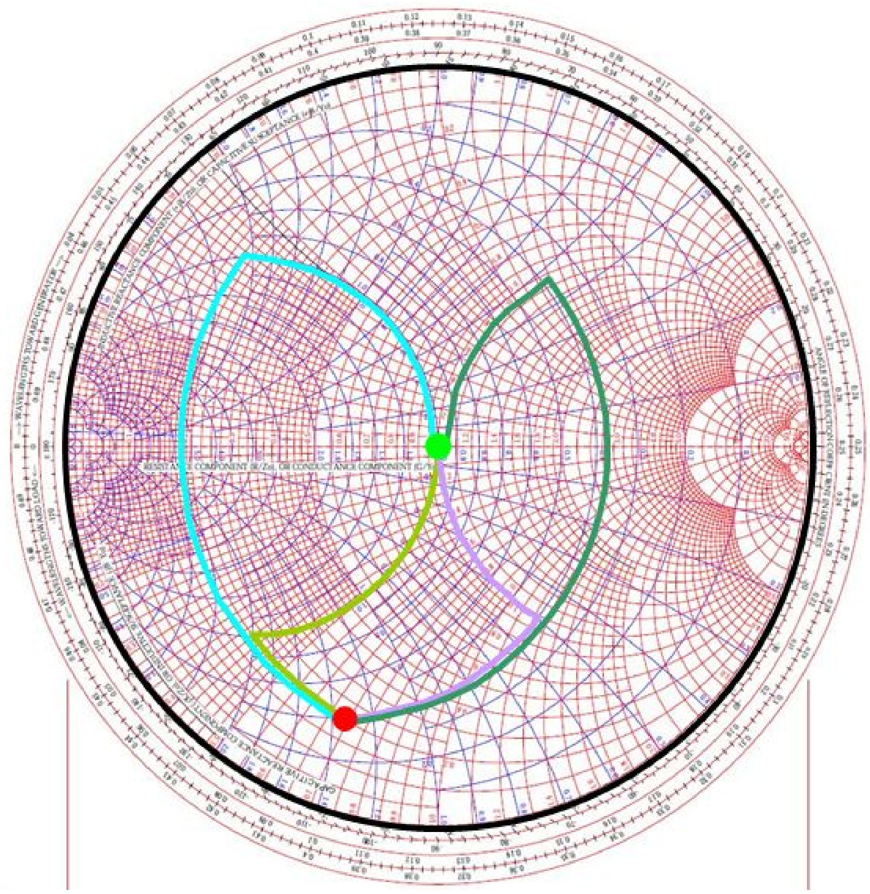

Figure 5: Visualize the tuning paths

of tuning components (C or L), we will use the following rules:

Smith's 1st rule Upward direction uses an inductor, downward direction uses a capacitor

Smith's 2nd rule Impedance tuning circle uses a series element, Admittance tuning circle uses a shunt element.

Smith's 3rd rule The paths passing through $R = \infty$ and $R = 0$ are are forbidden.

Regarding the Upward/downward rule, since some paths on a circular arc might, for instance, start upward but end up going downward, or vice-versa, it is the final direction, close to the end point of that tuning step, which determines the direction.

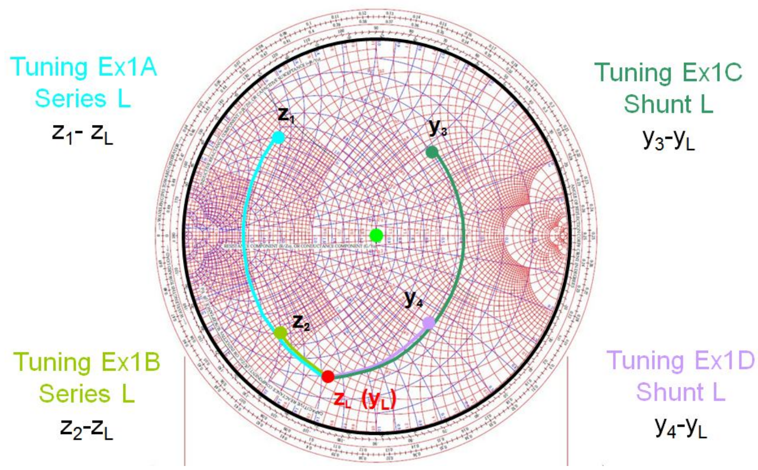

Figure 6: 1st tuning step

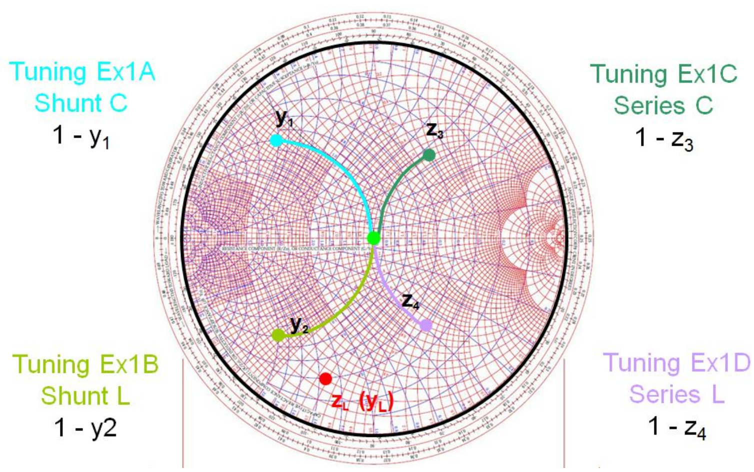

Figure 7: 2nd tuning step

In the figures (6), (7) above, the two tuning steps are shown separately. The reader should be able to confirm the tuning topology (series or shunt) and type of component (L or C) by using Smith's Three Rules. In figure (6), the Impedance or Admittance of Tuning Step 1 is labeled as Z_1, Z_2, Y_3, Y_4 on the Golden Circle. But in figure (7), the same points have been re-calculated as Y_1, Y_2, Z_3, Z_4 . This is because if Tuning Step 1 is an Impedance (Series) element, then Tuning Step 2 will be an Admittance (Shunt) element. Similarly, if Tuning Step 1 is an Admittance (Shunt) element, then Tuning Step 2 will be an Impedance (Series) element. To convert from Impedance to Admittance, and vice-versa, the equations are

Impedance from Admittance

$$R = \frac{G}{G^2 + B^2} \tag{20}$$

$$X = \frac{-B}{G^2 + B^2} \tag{21}$$

Admittance from Impedance

$$G = \frac{R}{R^2 + X^2} \tag{22}$$

$$B = \frac{-X}{R^2 + X^2} \tag{23}$$

10 A Complete Example

10.1 Z_{Load}

Let us start with a Z_{Load} = 0.3 + j0.96. There are four possible tuning circuits.

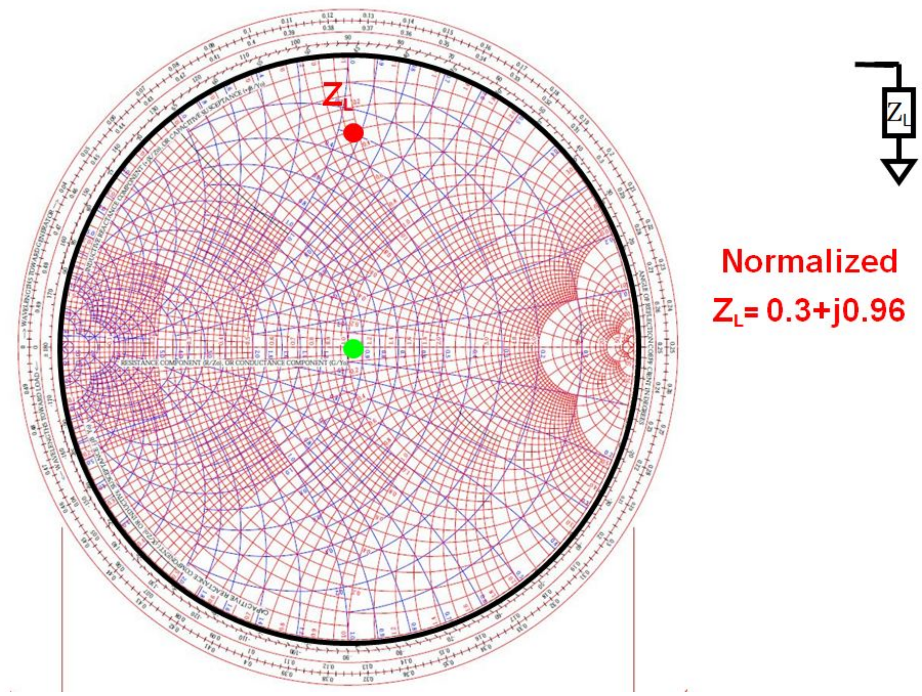

Figure 8: Z_{Load}

10.2 $C_{series}C_{shunt}$ Step 1

Impedance circle → Series element.
Downward movement → Capacitor.

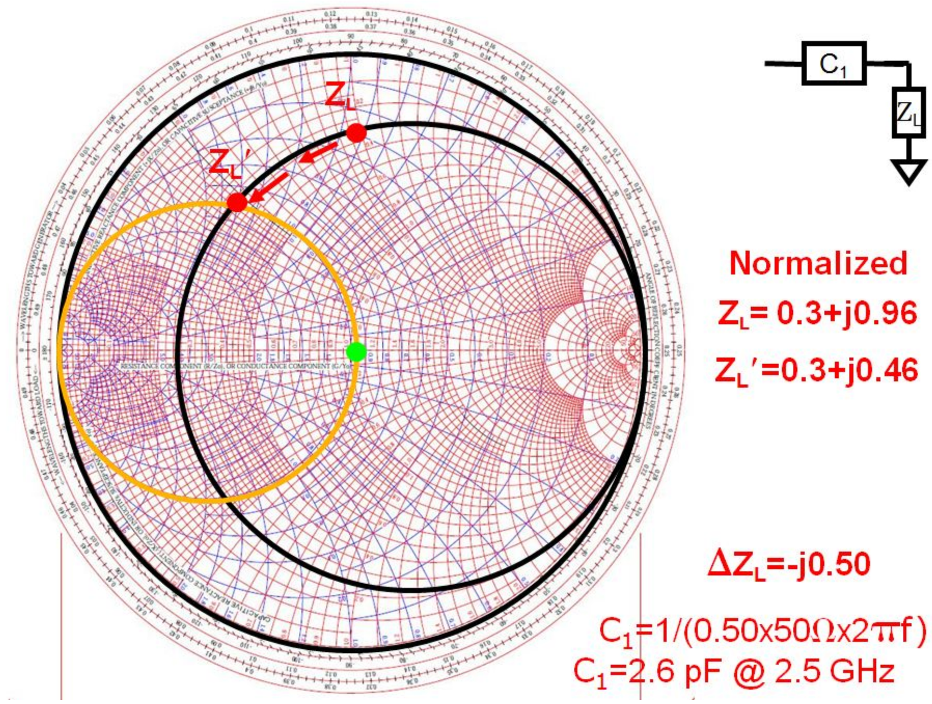

Figure 9: $Z_{Load} \rightarrow Z'_{Load}$

10.3 $C_{series}C_{shunt}$ Step 2

Admittance circle → Shunt element.
Downward movement → Capacitor.

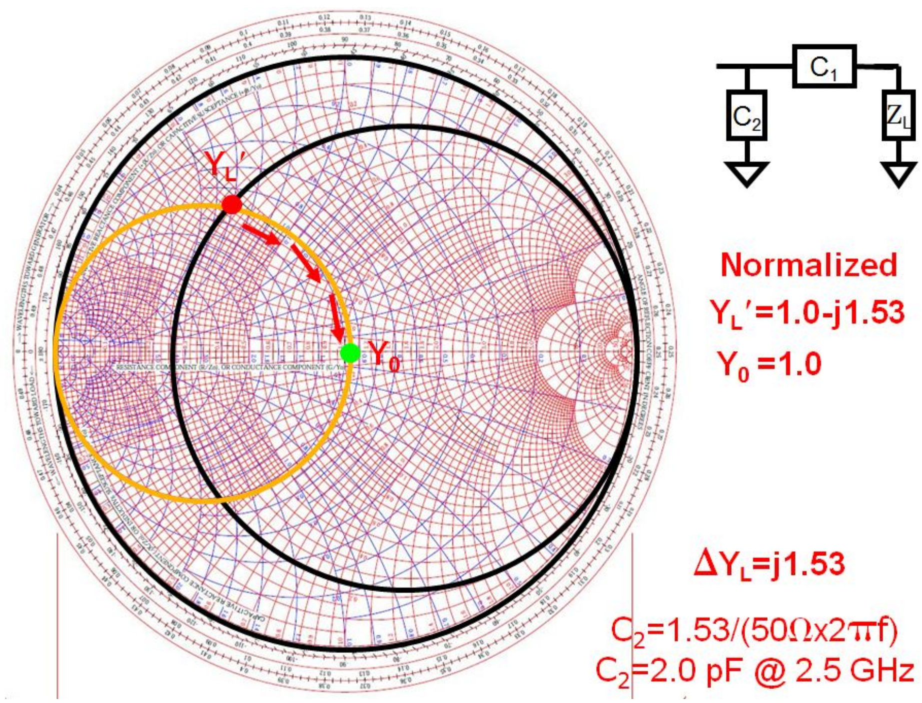

Figure 10: $Y'_{Load} \rightarrow 1$

10.4 $C_{series}L_{shunt}$ Step 1

Impedance circle → Series element.
Downward movement → Capacitor.

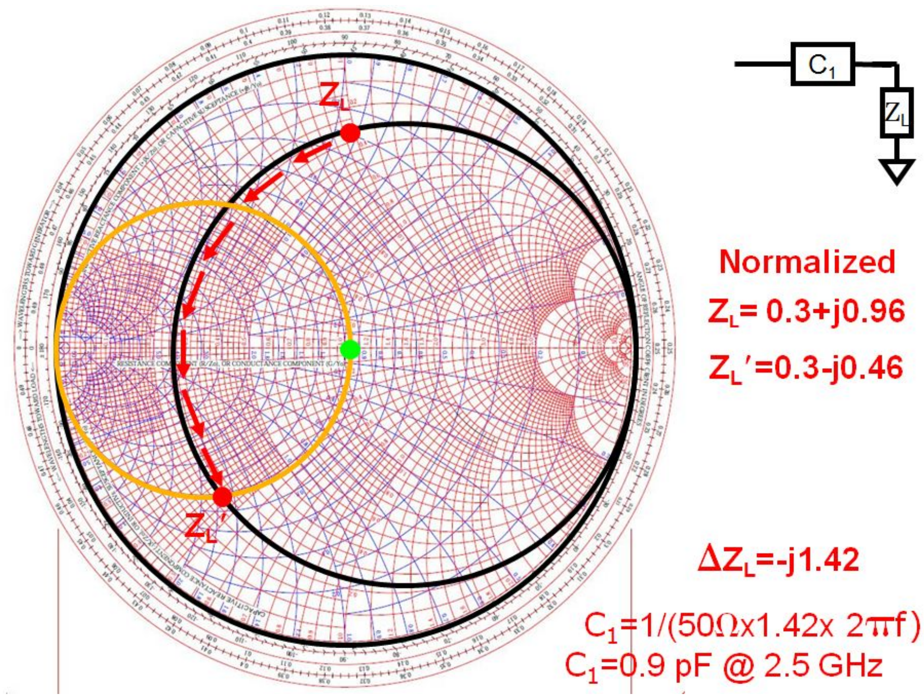

Figure 11: $Z_{Load} \rightarrow Z'_{Load}$

10.5 $C_{series}L_{shunt}$ Step 2

Admittance circle → Shunt element.
Upward movement → Inductor.

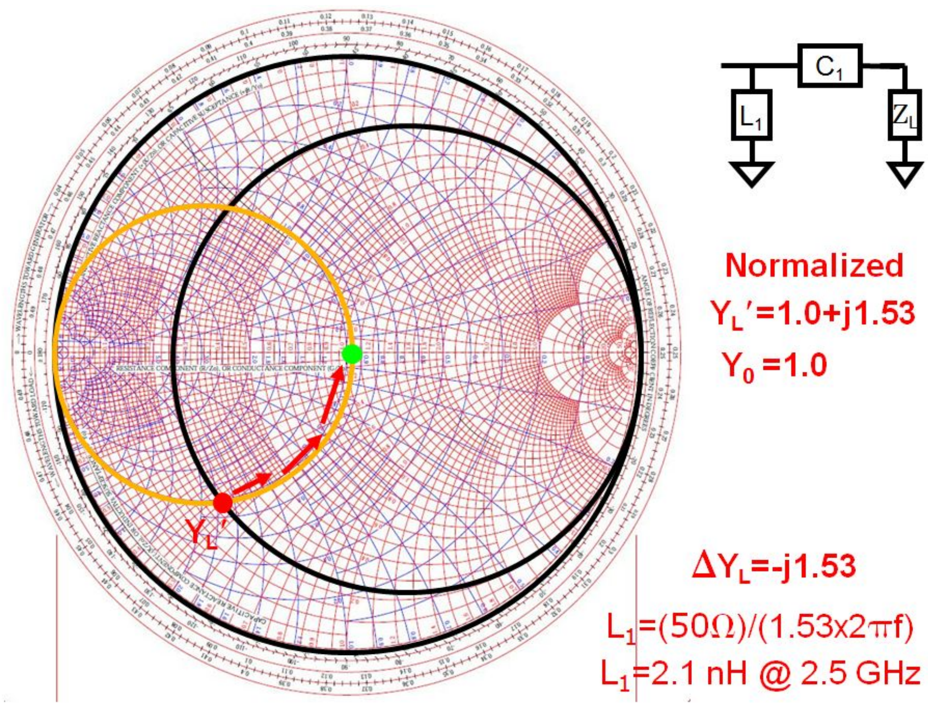

Figure 12: $Y'_{Load} \to 1$

10.6 $C_{shunt}C_{series}$ Step 1

Admittance circle → Shunt element.
Downward movement → Capacitor.

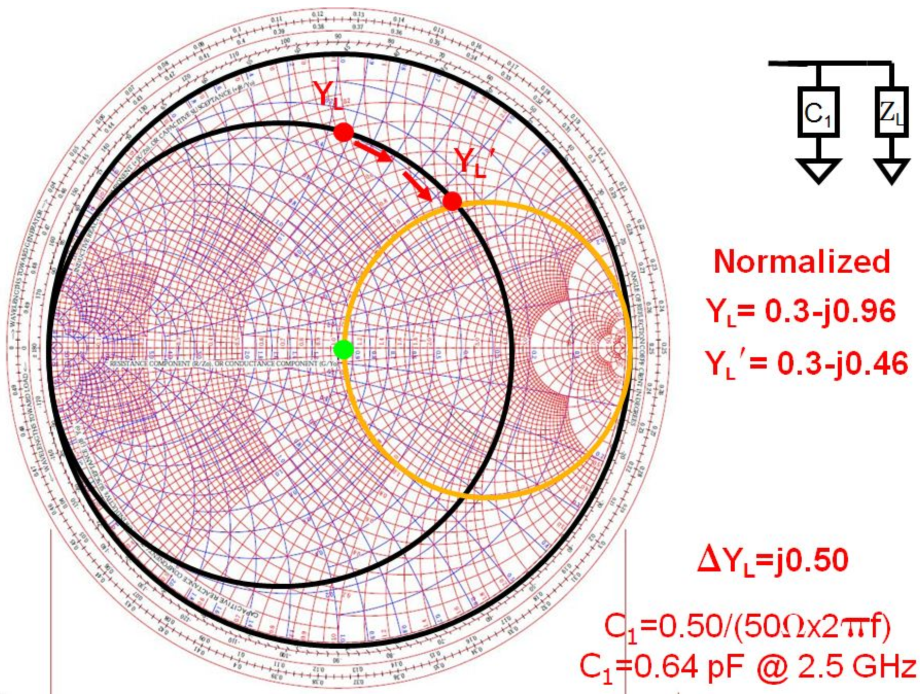

Figure 13: $Y_{Load} \rightarrow Y'_{Load}$

10.7 $C_{shunt} C_{series}$ Step 2

Impedance circle → Series element.
Downward movement → Capacitor.

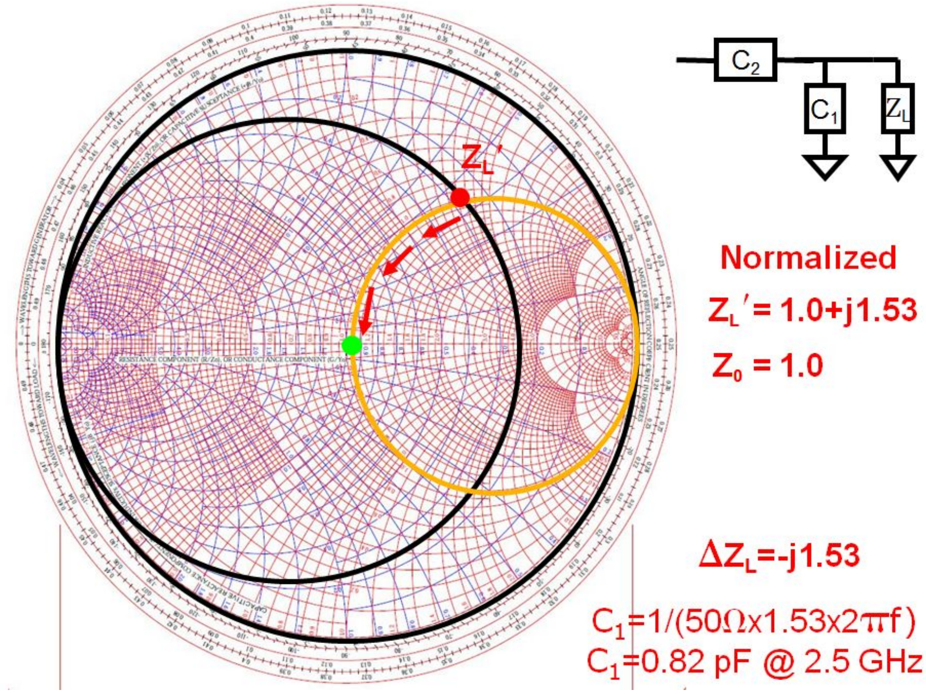

Figure 14: $Z'_{Load} \rightarrow 1$

10.8 $C_{shunt}L_{series}$ Step 1

Admittance circle → Shunt element.
Downward movement → Capacitor.

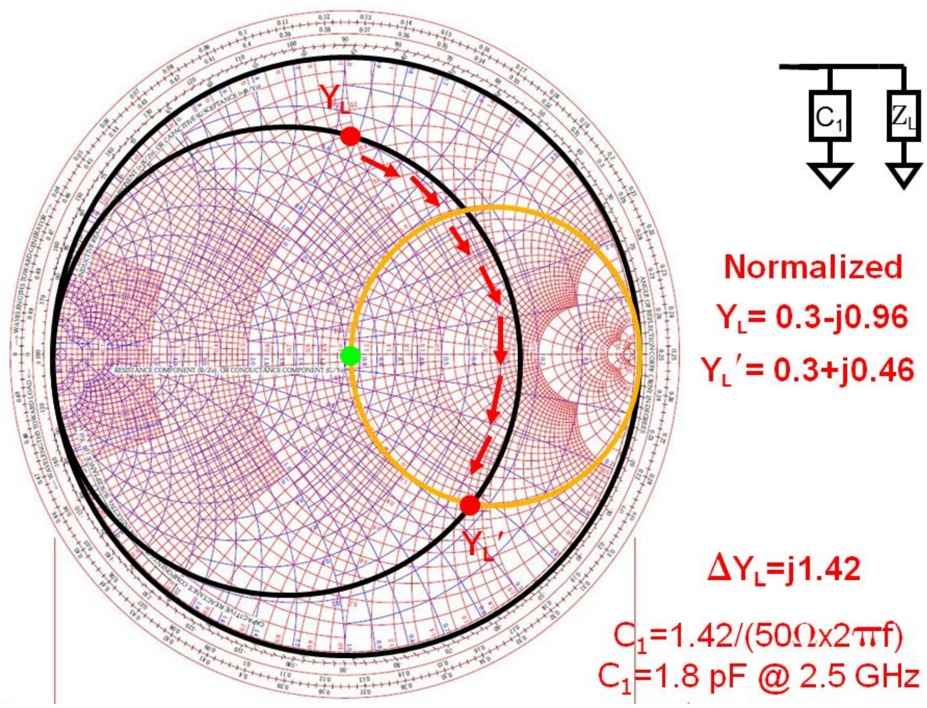

Figure 15: $Y_{Load} \rightarrow Y'_{Load}$

10.9 $C_{shunt}L_{series}$ Step 2

Impedance circle → Series element.
Upward movement → Inductor.

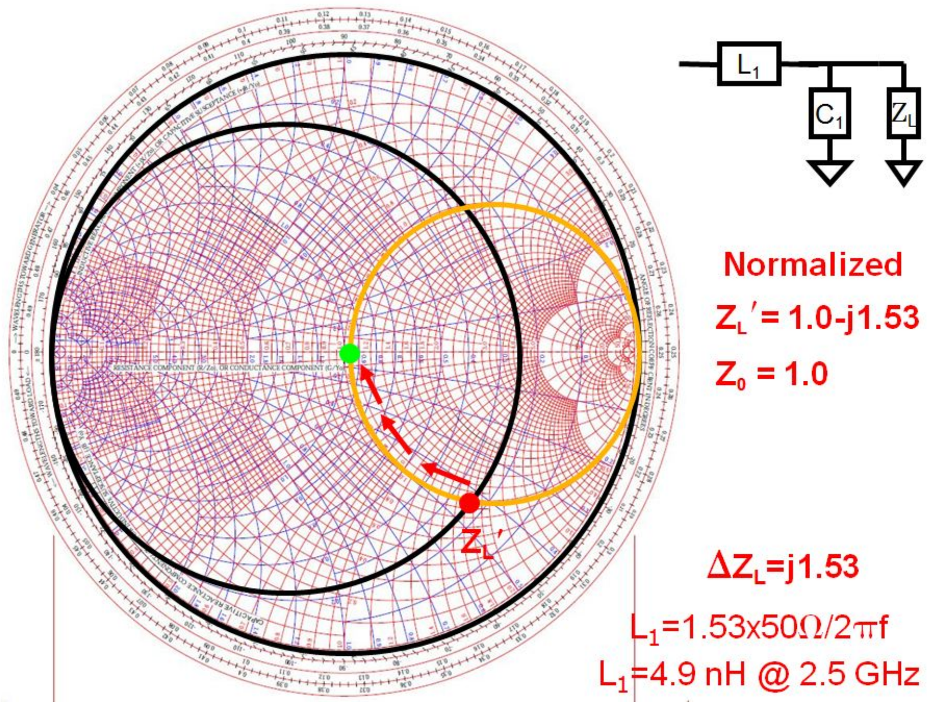

Figure 16: $Z'_{Load} \rightarrow 1$

11 Practical issues

In section (10), four different tuning circuits were found. Are all of these valid? In practice, no. One simple reason is that, although a circuit may be theoretically legitimate, the component values might not be reasonable. For instance, if the calculated C is 0.1 pF, which is at the limit of most capacitor tuning kits, it may be a poor choice. By the way, since we are on the topic of component values, the reader is encouraged to use inductor and capacitor kits with excellent range, tight tolerance, and tight spacing of electrical values. Murata makes such kits. [3].

S

0402 (50 VDC) Tight Tolerance

- Better Q and low ESR at VHF, UHF and microwave frequencies.
- 0201 and 0402 sizes with Copper inner electrode.
- Tight tolerance
  [W] = +/-0.05pF for 5pF & under [B] = +/-0.1pF for under 10pF
  [C] = +/-0.25pF for under 10pF
  [F] = +/-1% for 10pF & over [G] = +/-2% for 10pF & over
- Low power consumption, yield ratio improvement due to the better Q or lower ESR.
- GJM series is suited to VCO or PA module applications.
- GJM series is offered with a Ni barrier termination plated with Matte Tin (Sn), and is RoHS compliant.

GJM15-KIT-TTOL---E

No.	Description	Murata Global P/N	Qty.
1	0402/C0G/0.10pF/50V	GJM1555C1HR10WB01	20
2	0402/C0G/0.20pF/50V	GJM1555C1HR20WB01	20
3	0402/C0G/0.30pF/50V	GJM1555C1HR30WB01	20
4	0402/C0G/0.40pF/50V	GJM1555C1HR40WB01	20
5	0402/C0G/0.50pF/50V	GJM1555C1HR50WB01	20
6	0402/C0G/0.60pF/50V	GJM1555C1HR60WB01	20
7	0402/C0G/0.70pF/50V	GJM1555C1HR70WB01	20
8	0402/C0G/0.80pF/50V	GJM1555C1HR80WB01	20
9	0402/C0G/0.90pF/50V	GJM1555C1HR90WB01	20
10	0402/C0G/1.0pF/50V	GJM1555C1H1R0BB01	20
11	0402/C0G/1.1pF/50V	GJM1555C1H1R1BB01	20
12	0402/C0G/1.2pF/50V	GJM1555C1H1R2BB01	20
13	0402/C0G/1.3pF/50V	GJM1555C1H1R3BB01	20
14	0402/C0G/1.5pF/50V	GJM1555C1H1R5BB01	20
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19	0402/C0G/2.4pF/50V	GJM1555C1H2R4BB01	20
20	0402/C0G/2.7pF/50V	GJM1555C1H2R7BB01	20
21	0402/C0G/3.0pF/50V	GJM1555C1H3R0BB01	20
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23	0402/C0G/3.6pF/50V	GJM1555C1H3R6BB01	20
24	0402/C0G/3.9pF/50V	GJM1555C1H3R9BB01	20
25	0402/C0G/4.0pF/50V	GJM1555C1H4R0BB01	20
26	0402/C0G/4.3pF/50V	GJM1555C1H4R3BB01	20
27	0402/C0G/4.7pF/50V	GJM1555C1H4R7BB01	20
28	0402/C0G/5.0pF/50V	GJM1555C1H5R0BB01	20
29	0402/C0G/5.1pF/50V	GJM1555C1H5R1CB01	20
30	0402/C0G/5.6pF/50V	GJM1555C1H5R6CB01	20
31	0402/C0G/6.0pF/50V	GJM1555C1H6R0CB01	20
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38	0402/C0G/9.0pF/50V	GJM1555C1H9R0CB01	20
39	0402/C0G/9.1pF/50V	GJM1555C1H9R1CB01	20
40	0402/C0G/10pF/50V	GJM1555C1H100GB01	20
41	0402/C0G/12pF/50V	GJM1555C1H120GB01	20
42	0402/C0G/15pF/50V	GJM1555C1H150GB01	20
43	0402/C0G/18pF/50V	GJM1555C1H180GB01	20
44	0402/C0G/20pF/50V	GJM1555C1H200GB01	20

Figure 17: Murata 0402 Cap Kit

12 The Secret Tragedy of the Golden Circles

Up to this point, the Golden Circles have been portrayed as heroic entities, which guide the tuning paths to their final glorious destinations toward the center of the Smith Chart. But the Golden Circles possess an unfortunate feature as well. Let us now study a new and revealing tuning example.

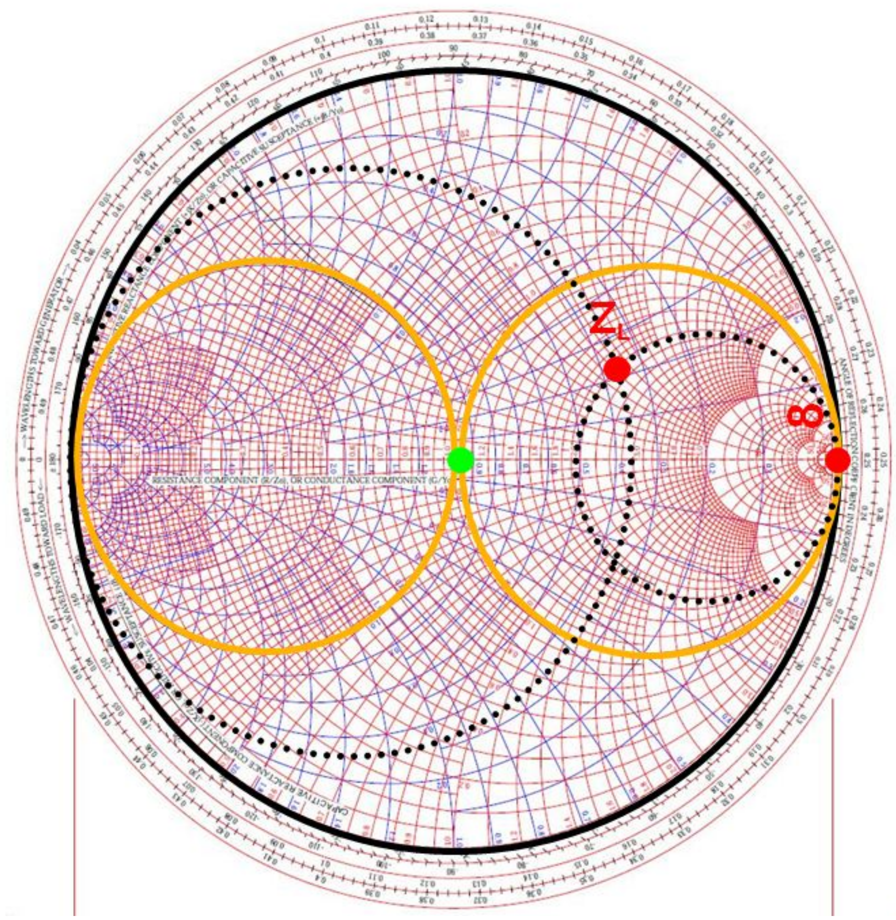

Figure 18: Z_{Load} inside Golden Circle

In figure (18) Z_{Load} is inside the Series Golden Circle. The dotted line circles are the Impedance and Admittance Tuning Circles which pass through Z_{Load}.

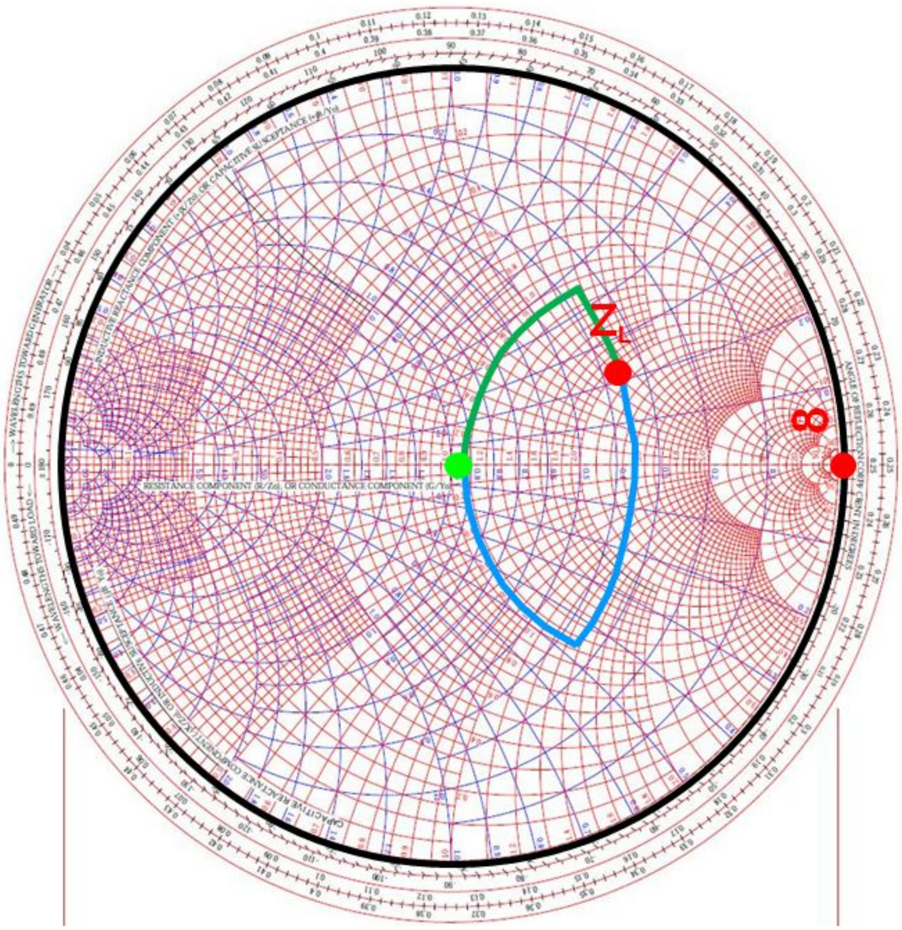

Figure 19: Z_{Load} inside Golden Circle

Now, let us visualize the possible tuning paths (figure (19)). In contrast to the example in section (10), there are only two possible tuning paths. A little thought should reveal to the reader why this is so. In the Two Step Tuning method, the first step moves along either an Impedance or Admittance circle to a Golden Circle. This implies that the two circles must *intersect*. But this Z_{Load} is *inside* the Impedance (Series) Golden Circle, so its Impedance tuning circle does not intersect that Golden Circle except at ∞ which is a forbidden tuning path. Only the Admittance tuning circle intersects a Golden Circle so the first tuning element can only be a shunt element.

> Insight Number 3A: A load point Z_{Load} which is inside or on a Golden Circle will only have 2 possible tuning paths.

> Insight Number 3B: A load point Z_{Load} which is outside the Golden Circles will have 4 possible tuning paths.

This loss of tuning paths inside the Golden Circle explains one of the *Great Mysteries of Tuning* which many readers have personally experienced. It is the frustrating inability to achieve match after choosing a particular topology. If the reader experiences this condition again, it will now be accompanied by the peaceful and enlightened thought, "Ah, I am inside a Golden Circle!".

13 Mr. Smith answers doubting readers

Although the methodology described so far in this article is based on rigorous mathematics, many readers will remain skeptical based on extensive experience. The obvious objection to Mr. Smith's method is that changing a component, for instance changing a series inductor from L_1 to L_2, will follow *neither* an Impedance (Series) tuning circle, *nor* an Admittance (Shunt) tuning circle on the Network Analyzer display. It just goes in a random direction, and only by tedious trial and error will match be achieved. Here is an example:

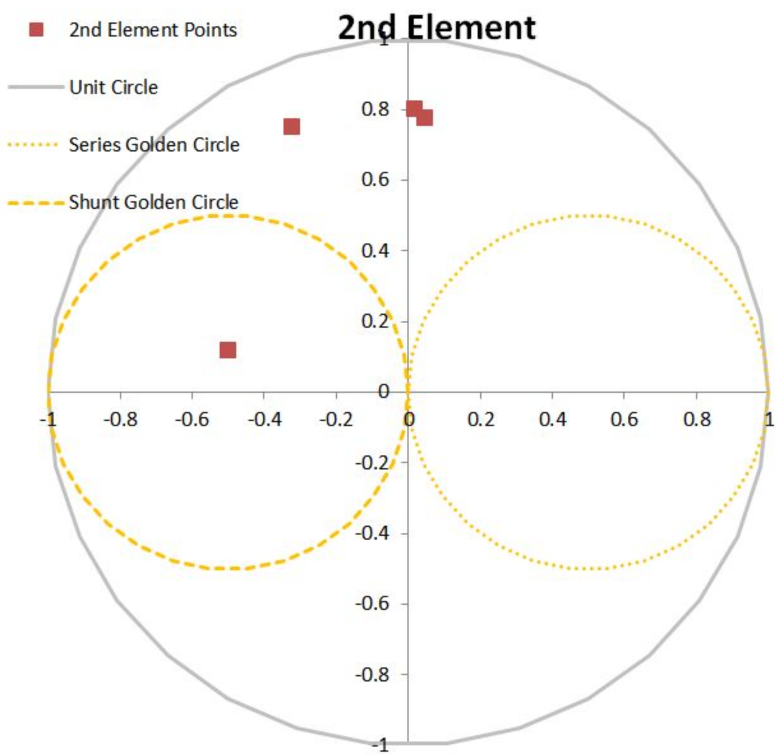

Figure 20: Random movement?

The four plotted points are L_1 = 3.3 nH, L_2 = 5.6 nH, C_1 = 3.0 pF, C_2 = 5.6 pF, for a Step 2 Shunt Element tuning attempt. This is what a Network Analyzer showed after its initial calibration at the end of the RF cable. Reference Plane Extension is 0 pS, and gain is 1.0.

Mr. Smith's simple answer to this problem is *Reference Plane Extension* and *Loss*.

Now consider the next graph, which shows the same four points at Reference Plane Extension 102 pS, and with a gain of 1.2 (the polar magnitude of the four points is multiplied by 1.2)

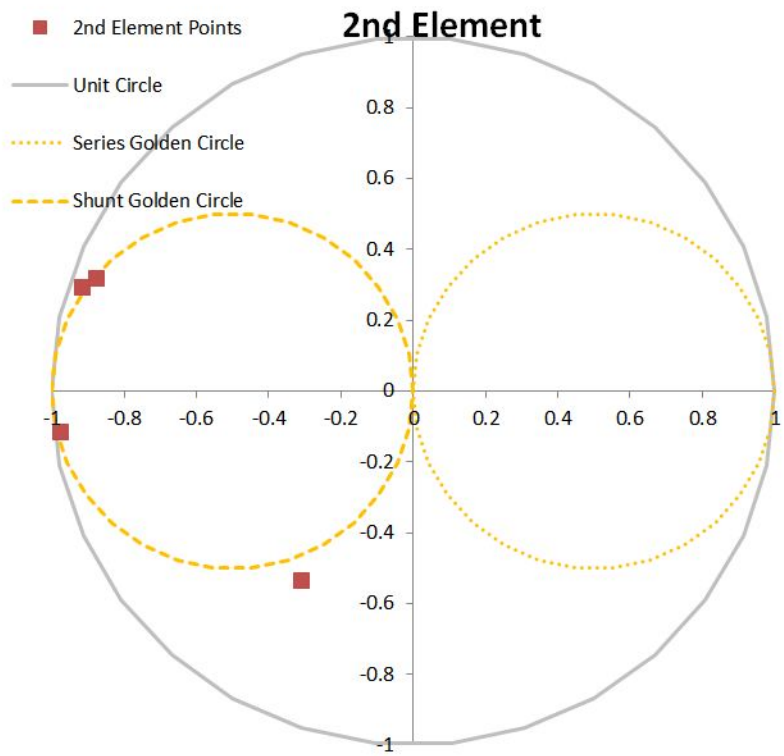

Figure 21: With Reference Plane Extension and Gain

The points do not line up perfectly with the Golden Circle, due to component parasitics, layout parasitics, and even soldering repeatability issues. Their exact angular position on the Golden Circle also does not line up with the theoretical Smith Chart grid. And the Reference Plane Extension may not indicate the real physical location but could be partially compensating for a layout parasitic. Despite all these negative features, the key point is that their movement will indeed tend to follow Smith's rules. Although imperfect, the behavior is good enough to follow intuitive reasoning based on Smith's Rules. A more detailed discussion of this will be given in Part II to be published later. However, by now, the reader is probably ready for a long quiet vacation. But let us introduce our last, strongly recommended bit of advice which is easy to implement:

Mr. Smith's Strong Recommendation Always record the raw data of tuning values and their Γ_{11} at a fixed Reference Plane Extension

As more data is accumulated this data clearly will reveal the orientation and position of the Impedance and Admittance circles.

References

[1] Agilent Technologies, *AN 154: S-Parameter Design,* 1990.

[2] K. Kurokawa, *Power Waves and the Scattering Matrix,* IEEE Transactions on Microwave Theory and Techniques, Vol. MTT-13, No. 2, March, 1965.

[3] Murata, *Design Engineering Kit Catalog No. G-06-G,* 2010