Hideo Okawara's
Mixed Signal Lecture Series

DSP-Based Testing – Fundamentals 24
AM Waveform Generation

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Preface to the Series

ADC and DAC are the most typical mixed signal devices. In mixed signal testing, analog stimulus signal is generated by an arbitrary waveform generator (AWG) which employs a D/A converter inside, and analog signal is measured by a digitizer or a sampler which employs an A/D converter inside. The stimulus signal is created with mathematical method, and the measured signal is processed with mathematical method, extracting various parameters. It is based on digital signal processing (DSP) so that our test methodologies are often called DSP-based testing.

Test/application engineers in the mixed signal field should have thorough knowledge about DSP-based testing. FFT (Fast Fourier Transform) is the most powerful tool here. This corner will deliver a series of fundamental knowledge of DSP-based testing, especially FFT and its related topics. It will help test/application engineers comprehend what the DSP-based testing is and assorted techniques.

Editor's Note

For other articles in this series, please visit the Verigy web site at
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Preface

In mixed signal device testing, various modulated signals are required as well as sinusoids and multi-tones as stimulus signals, which are programmed and generated by arbitrary waveform generators (AWG). Amplitude, phase and frequency modulations are the most typical modulation. There are some methods available to create modulated waveform. In the next few issues of go/semi we will discuss how to create modulated signals. AM waveform generation is the topic this month.

Signal Modulation

A simple signal $v(t)$ can be defined as below;

$$v(t) = A \cos(\omega t + \phi) \tag{1}$$

where A is amplitude, ω is angular frequency and ϕ is phase offset. Each one of the parameters can be modified along time and carry information. So in general modulated signal can be expressed as below;

$$v(t) = A(t) \cos(\omega(t)t + \phi(t)) \tag{2}$$

Each parameter can be a function of time. When the amplitude A is modulated, it is called amplitude modulation (AM). When the phase ϕ is modulated, it is called phase modulation (PM). When the (angular) frequency ω is modulated, it is called frequency modulation (FM).

AM Waveform Generation – Direct Method

AM is the topic of this month. It is defined as Equations (3) and (4) where A is the carrier amplitude, $u(t)$ is the modulation signal. In the equations using cosine and sine has no special intention. You may add any static phase offset in them, if necessary.

$$
\begin{aligned}
v(t) &= A\cos(\omega_p t) \\
&= A(1 + u(t))\cos(\omega_p t)
\end{aligned} \tag{3}
$$

$$u(t) = R\sin(\omega_q t) \tag{4}$$

If $u(t)=0$ that is no information, the signal $v(t)$ is continuous sinusoid. When $u(t)$ is stimulated, the signal amplitude is modified by the trend of $u(t)$. This is the most typical AM. It can be coded straightforward as List 1. A test signal waveform programmed in an AWG is usually looped many times while tests would be performed. For this purpose, you must create a whole number cycles of sinusoids in the unit waveform period (UWP) of the AWG. In List 1, 801 cycles of sinusoid is programmed as the carrier, and 9 cycles of sinusoidal modulation signal is programmed. The waveforms created by List 1 are illustrated in Figure 1, where (a) is the carrier signal, (b) is the modulation signal, and (c) is the final modulated signal. As you can see the envelope of (c) contains the modulation signal information that is R and ω_q.

```
01:    INT     i,Nawg,Np,Nq;
02:    DOUBLE  dFawg,dFp,dFq,dP,dQ,dR,dA,dU;
03:    ARRAY_D dwave;
04:
05:    dFawg=102.4 MHz;                    // AWG Sampling Rate
06:    Nawg=8192;                         // AWG Data Size
07:    dFresln=dFawg/Nawg;                // Frequency Resolution
08:
09:    dFp=10.0125 MHz;                   // Carrier Frequency
10:    Np=801;                           // # of cycles in the UTP
11:    dFq=112.5 kHz;                     // Modulation Frequency
12:    Nq=9;                             // # of cycles in the UTP
13:
14:    dA=0.5;                           // Carrier Amplitude [V]
15:    dR=1.0;                           // Modulation Index (1.0=100%)
16:    dP=2.0*M_PI*Np/Nawg;              // Instantaneous Phase (Carrier)
17:    dQ=2.0*M_PI*Nq/Nawg;              // Instantaneous Phase (Mod)
18:
19:    dwave.resize(Nawg);                // AM Wave Container
20:    for (i=0;i<Nawg;i++) {
21:      dU=dR*sin(dQ*i);                // Modulation Signal
22:      dwave[i]=dA*(1.0+dU)*cos(dP*i); // AM Modulated Signal
23:    }
```

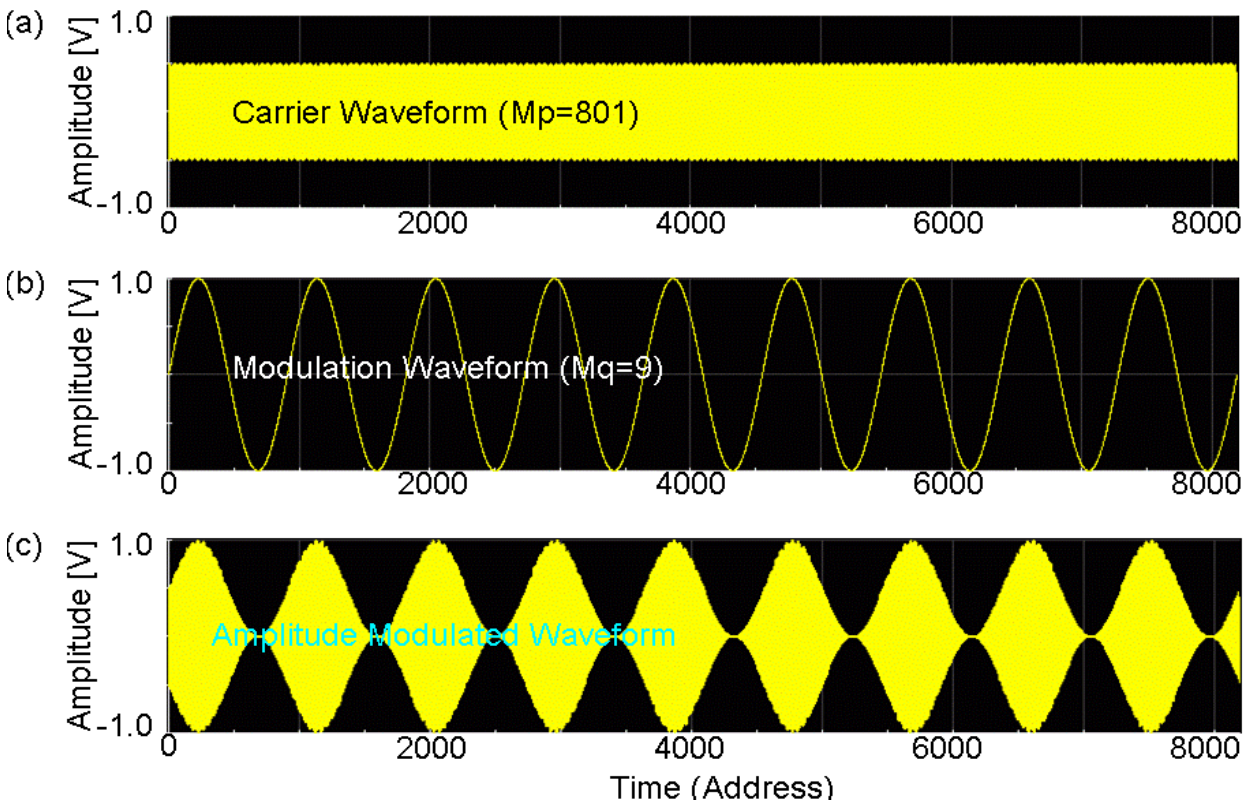**List 1:   AM Program – Straightforward**

Figure 1:   AM Signal by List 1

By performing FFT to the AM signal, its frequency spectrum is shown in Figures 2 (b) and its close-up (c). AM signal is constructed by the carrier, lower side band (LSB) and upper side band (USB). The bin number of LSB and USB is the carrier's bin number +/-modulation signal cycles in the UWP.
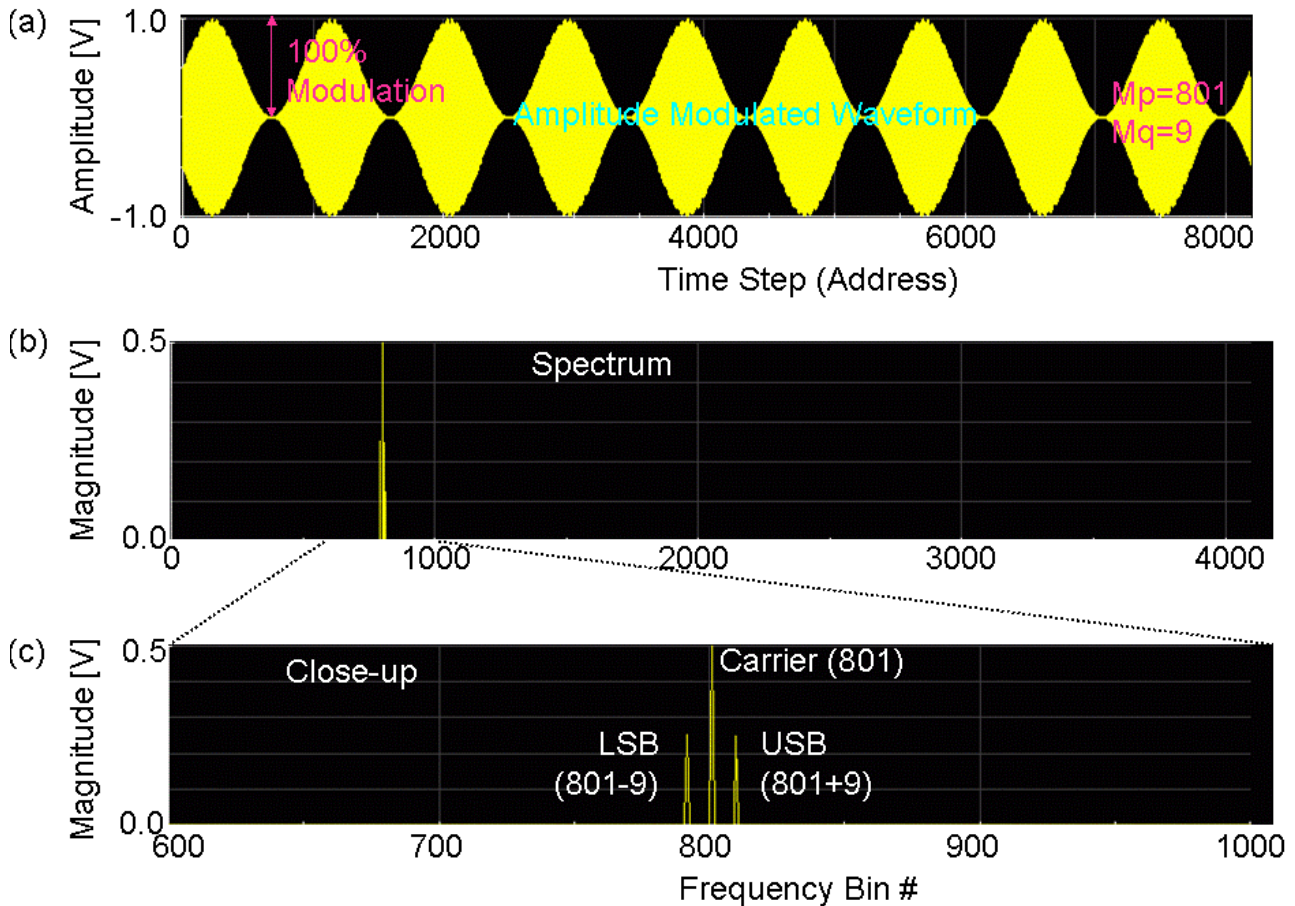

Figure 2: Spectrum of AM Signal

AM Waveform Generation – IFFT Method

Inserting a static phase offset θ in the modulation signal $u(t)$, Equation (3) with (4) can be expanded as follows;

$$
\begin{aligned}
v(t) &= A\big(1 + R\sin(\omega_q t + \theta)\big)\cos(\omega_p t) \\
&= A\big(\cos(\omega_p t) + R\sin(\omega_q t + \theta)\cos(\omega_p t)\big) \\
&= A\left(\cos(\omega_p t) + \frac{R}{2}\big(\sin((\omega_p + \omega_q)t + \theta) + \sin((\omega_p - \omega_q)t - \theta)\big)\right)
\end{aligned}
\tag{5}
$$

The third line of Equation (5) shows the carrier, USB and LSB components so that it supports the frequency spectrum in Figure 2(c).

Looking at Equation (5), you may notice that AM signal can be programmed in the frequency domain. The point is that there are three components – the carrier, USB and LSB. The Level of

side-bands has the factor of 1/2. The phase offset of the modulation signal appears opposite polarity between the USB and LSB. Figure 3(a) illustrates the spectrum design. The magnitude of the carrier, USB and LSB is set 0.5 and 0.25 respectively. The USB has a static phase offset, for example, $\pi/4$ here so that the opposite polarity phase is assigned to LSB.
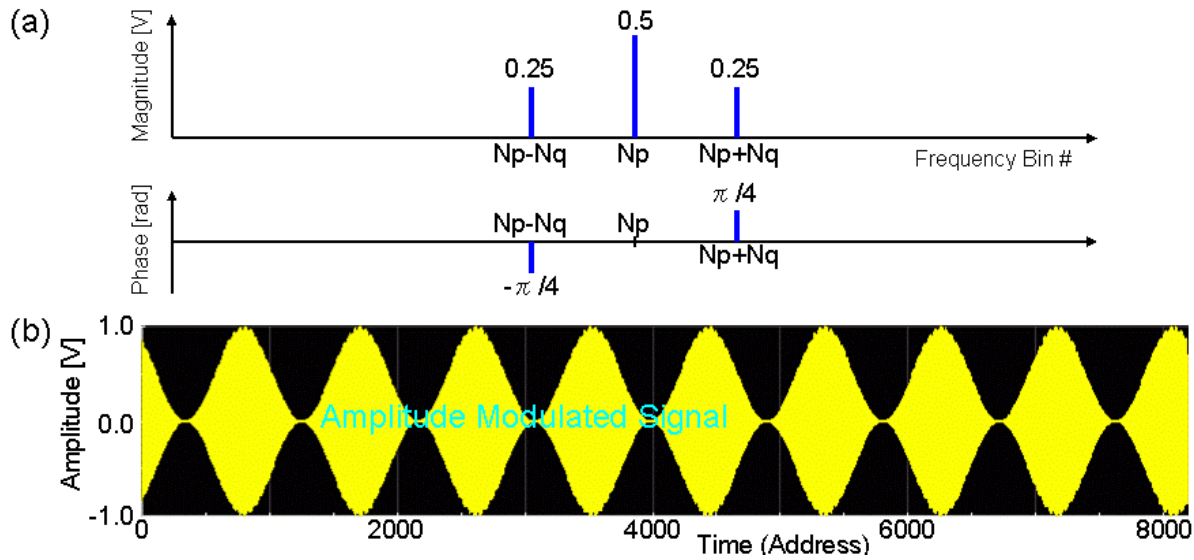

Figure 3:   AM Design in Frequency Domain

```
01:    INT             i,Nawg,Np,Nq;
02:    DOUBLE          dFawg,dFp,dFq;
03:    ARRAY_D         dWave,dPha,dAmp;
04:    ARRAY_COMPLEX CSp,CWave;
05:
06:    Nawg=8192;                          // AWG Data Size
07:    Np=801;                             // # of cycles in the UTP
08:    Nq=9;                               // # of cycles in the UTP
09:
10:    dAmp.resize(Nawg/2); dAmp.init(0.0); // Amplitude Initialize
11:    dPha.resize(Nawg/2); dPha.init(0.0); // Phase Initialize
12:    dAmp[Np]=0.5;                       // Carrier Amplitude
13:    dPha[Np]=0.0;                       // Carrier Phase
14:    dAmp[Np+Nq]=0.25;                   // USB Amplitude
15:    dPha[Np+Nq]=M_PI/4.0;               // USB Phase
16:    dAmp[Np-Nq]=dAmp[Np+Nq];            // LSB Amplitude = USB
17:    dPha[Np-Nq]=-dPha[Np+Nq];           // LSB Phase = Opposite of USB
18:    DSP_POL_RECT(dAmp,dPha,CSp);        // Polar to Rectangular Conv.
19:
20:    CSp.resize(Nawg);                   // Spectrum Container Full-size
21:    for (i=1;i<(Nawg/2);i++)            //
22:       CSp[Nawg-i]=Conjugate(CSp[i]);   // Complex Conjugate
23:    CSp(Nawg/2)=CZero();                // Complex Zero
24:    DSP_IFFT(CSp,CWave);                // IFFT
25:    dWave.resize(Nawg);                 // Waveform Container
26:    dWave=CWave.getReal();              // Get real part
27:    DSP_MUL_SCL(0.5,dWave,dWave);       // Scaling half
```
List 2:   **AM Program: Design in Frequency Domain**

   List 2 is the program code based on Figure 3(a). By performing IFFT at Line 24, the AM signal is generated as Figure 3(b).

   Looking at Equation (5) and Figure 2(a), the carrier does not convey any information so that AM system could omit to send the carrier component. In this case the name of "carrier" has nonsense. When both LSB and USB are used, it is called DSB or double sideband AM. When either LSB or USB is used, it is called SSB or single sideband AM.

```
01:    INT           i,Nawg,Np,Nq;           // No Carrier DSB
02:    DOUBLE        dFawg,dFp,dFq;
03:    ARRAY_D       dWave,dPha,dAmp;
04:    ARRAY_COMPLEX CSp,CWave;
05:
06:    Nawg=8192;                             // AWG Data Size
07:    Np=801;                                // # of cycles in the UTP
08:    Nq=9;                                  // # of cycles in the UTP
09:
10:    dAmp.resize(Nawg); dAmp.init(0.0); // Amplitude Container Initialize
11:    dPha.resize(Nawg); dPha.init(0.0); // Phase Container Initialize
12:    dAmp[Np+Nq]=0.5;                       // Define Amplitude of USB
13:    dPha[Np+Nq]=M_PI/4.0;                  // Define Phase of USB
14:    dAmp[Np-Nq]=dAmp[Np+Nq];               // Define LSB as
15:    dPha[Np-Nq]=-dPha[Np+Nq];              //   Complex Conjugate of USB
16:    DSP_POL_RECT(dAmp,dPha,CSp);           // Polar to Rectangular Conv.
17:
18:    CSp.resize(Nawg);                      // Spectrum Container Full-size
19:    for (i=1;i<(Nawg/2);i++)               //
20:        CSp[Nawg-i]=Conjugate(CSp[i]);     // Complex Conjugate
21:    CSp[Nawg/2]=CZero();                   // Complex Zero
22:    DSP_IFFT(CSp,CWave);                   // IFFT
23:    dWave.resize(Nawg);                    // Waveform Container
24:    dWave=CWave.getReal();                 // Get real part
25:
```
List 3:   **DSB Program Example**

List 3 is the code example of DSB. Figure 4 is the result that the program List 3 generates. As you can see, the amplitude of LSB and USB is doubled compared to Figure 2(c). This is because the carrier is not included in the signal. So DSB is more efficient than regular AM. SSB is more efficient than DSB. If you remove Lines 14 and 15 in List 3, you can make the amplitude 1.0 at Line 12. Then the SSB spectrum and waveform look as Figure 5. This is just a single tone of (Np+Nq) cycles. For demodulating DSB and SSB signals, you have to provide the carrier signal by yourself.

$$R\sin\big((\omega_p + \omega_q)t + \theta\big)\cdot\cos(\omega_p t) = \frac{R}{2}\big\{\sin\big((2\omega_p + \omega_q)t + \theta\big) + \sin\big(\omega_q t + \theta\big)\big\} \tag{6}$$

In Equation (6), the carrier $\cos(\omega_p t)$ is multiplied to the USB signal, and the result splits into two components – one is very high frequency component and the other is low frequency components which corresponds to the modulation signal. The low frequency component can be separated by low pass filtering operation. This processing is called "product detection" and it is popular demodulation method in DSB and SSB.
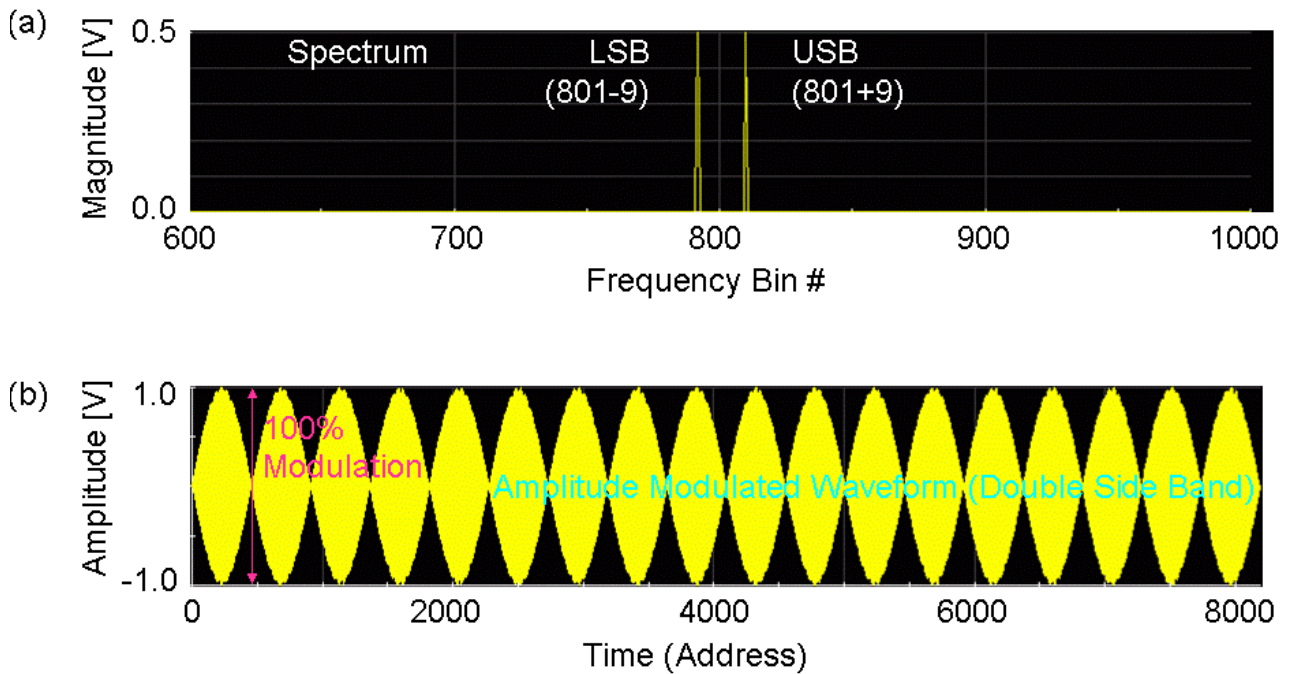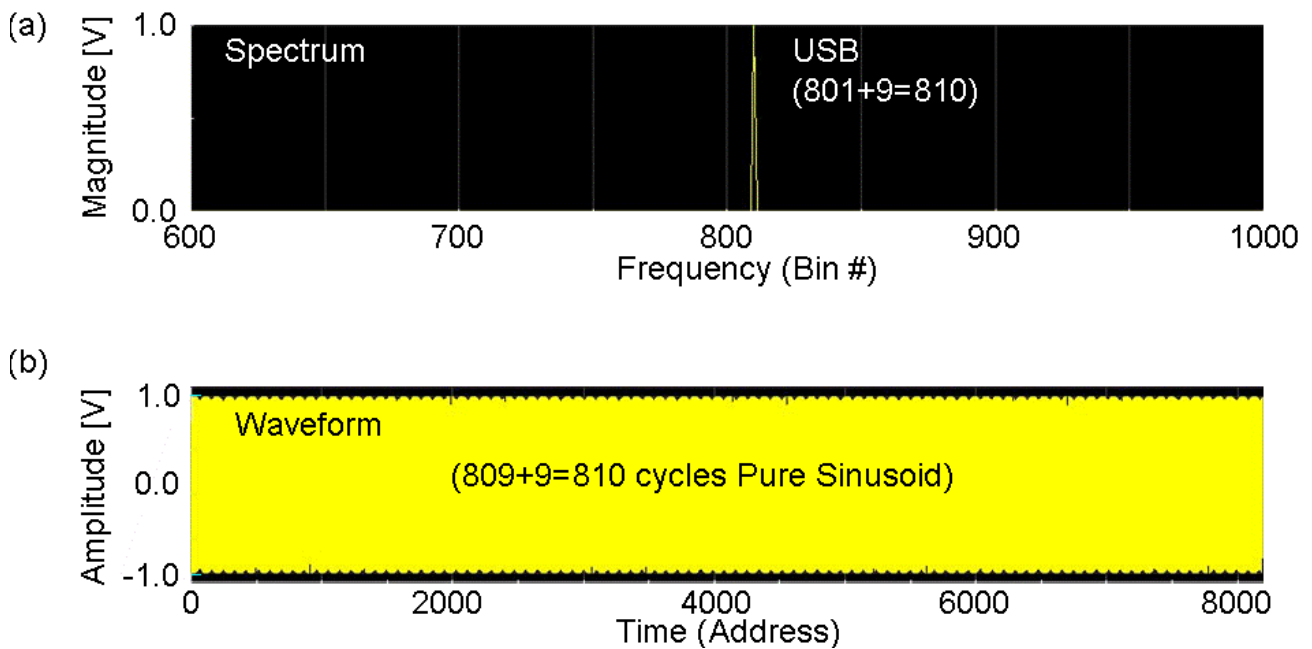
Figure 4   DSB Spectrum and Waveform

Figure 5   SSB Spectrum and Waveform